## A Theoretical Framework for Symbolic Quick Error Detection



Florian Lonsing<br>Subhasish Mitra<br>Clark Barrett

Paper published at Formal Methods in Computer-Aided Design (FMCAD) 2020 Preprint: https://arxiv.org/abs/2006.05449

## Context: Pre-Silicon Verification

## Electronic Design Automation (EDA)

## Design

 (HDL)

- Model checking vs. non-formal simulation or testing.


## Context: Pre-Silicon Verification

## Electronic Design Automation (EDA)

## Design

 (HDL)

- Model checking vs. non-formal simulation or testing.

Symbolic Quick Error Detection (SQED)



Is M self-consistent?


Cf. traditional MC:
Property set (spec)

- No formal spec or implementation-specific properties.
- Self-consistency: universal property.
- Industrial-strength technique.


## Our Contributions: SQED Soundness Proof



- If $M$ not self-consistent then $B \in M$.

No spurious cex

## Our Contributions: SQED Completeness Proof



- If $B \in M$ then $M$ not self-consistent.


## Conditional/full Completeness

## Self-Consistency

- Processor Design M:
$i_{1}, i_{2}, \ldots, i_{n}+$ inputs (regs/mem)
$i_{1}{ }^{\prime}, i_{2}{ }^{\prime}, \ldots, i_{n}{ }^{\prime}+$ inputs' (regs/mem)


outputs (regs/mem)<br>outputs' (regs/mem)

## Self-Consistency

- Processor Design M:
$i_{1}, i_{2}, \ldots, i_{n}+$ inputs (regs/mem)
$i_{1}{ }^{\prime}, i_{2}{ }^{\prime}, \ldots, i_{n}{ }^{\prime}+$ inputs' (regs/mem)


HW designs have complex internal state (pipeline,...).

## Formal Model of Processors and SQED



- State $s_{0}$ : mapping from locations $\mathcal{L}$ to values.
- $\mathcal{L}$ : register and memory locations.


## Formal Model of Processors and SQED



- Executing instruction $i$ : read from input locations in $s_{0}$. - Update output location in $s_{1}$ by opcode.


## Formal Model of Processors and SQED

| $\boldsymbol{L}_{\boldsymbol{O}}$ |
| :--- |
| $\boldsymbol{L}_{\boldsymbol{D}}$ |

- Original locations $\mathcal{L}_{\boldsymbol{o}}$.
- Duplicate locations $\mathcal{L}_{\boldsymbol{D}}$.
- Arbitrary, fixed bijective mapping $\mathrm{L}_{\mathrm{D}}: \mathcal{L}_{\boldsymbol{O}} \rightarrow \boldsymbol{\mathcal { L }}_{\boldsymbol{D}}$.


## Formal Model of Processors and SQED



- Original instruction $i_{o}$ : read/write $\mathcal{L}_{O}$ only.
- Duplicate $i_{D}$ : same opcode, read/write $\mathcal{L}_{\boldsymbol{D}}$ only.


## Formal Model of Processors and SQED



- $\boldsymbol{i}_{O}=i_{O, 1}, \ldots, i_{O, n}, \boldsymbol{i}_{D}=i_{D, 1}, \ldots, i_{D, n}, \boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right)$.
- QED test: concatenation $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$ of $2 n$ instructions.


## Formal Model of Processors and SQED

QEDcons $\left(s_{0}\right)$


$$
s_{0}\left(\mathcal{L}_{O}\right)=s_{0}\left(\mathcal{L}_{D}\right)
$$

$Q E D \operatorname{cons}\left(s_{2 n}\right)$ ?


$$
s_{2 n}\left(\mathcal{L}_{O}\right)=s_{2 n}\left(\mathcal{L}_{D}\right) ?
$$

- $\boldsymbol{i}_{O}=i_{O, 1}, \ldots, i_{O, n}, \boldsymbol{i}_{D}=i_{D, 1}, \ldots, i_{D, n}, \boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right)$.
- QED test: concatenation $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$ of $2 n$ instructions.
- QED-consistent state: matching values at $\mathcal{L}_{\boldsymbol{O}}$ and $\mathcal{L}_{\boldsymbol{D}}$.


## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

$$
n=1
$$

Select $n$ original instructions $\boldsymbol{i}_{0}$

## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

$$
n=1
$$



## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

$$
n=1
$$

Get $n$ duplicate instructions

$$
\boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right) \text { using } L_{D}
$$

$$
\text { QED test } \boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}
$$

Model: execute $\boldsymbol{i}$ (length $2 n$ ) in QED-consistent initial state $s_{0}$

## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

$$
n=1
$$

Get $n$ duplicate instructions

$$
\boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right) \text { using } L_{D}
$$

$$
\text { QED test } \boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}
$$

Model: execute $\boldsymbol{i}$ (length $2 n$ ) in QED-consistent initial state $s_{0}$

## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$
and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

Counterexample $\boldsymbol{i}$

$$
n=1
$$

Select $n$ original instructions $\boldsymbol{i}_{O}$

Get $n$ duplicate instructions

$$
\boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right) \text { using } L_{D}
$$

$$
\text { QED test } \boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}
$$

Model: execute $\boldsymbol{i}$ (length $2 n$ ) in QED-consistent initial state $s_{0}$

## Using BMC in SQED

Select partition $\mathcal{L}_{O}, \mathcal{L}_{D}$ of $\mathcal{L}$ and mapping $L_{D}: \mathcal{L}_{O} \rightarrow \mathcal{L}_{D}$

Counterexample $\boldsymbol{i}$

Continue using $\mathcal{L}_{O}, \mathcal{L}_{D}, L_{D}$ ?

Get $n$ duplicate instructions

$$
\boldsymbol{i}_{D}=\operatorname{Dup}\left(\boldsymbol{i}_{O}\right) \text { using } L_{D}
$$

QED test $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$
Model: execute $\boldsymbol{i}$ (length $2 n$ ) in QED-consistent initial state $s_{0}$




## Processor Correctness

## Processor P is correct:

- $\forall s \in S, i \in I . \operatorname{reach}(s) \rightarrow \operatorname{Spec}(s, i, T(s, i))$.
- All instructions execute correctly in all reachable states.

Abstract specification Spec (theory only):

1. Correct output value product.
2. All non-output locations unchanged.

## Bugs

## Reachable states

Initia states


## Soundness of SQED

$$
Q E D \operatorname{cons}\left(s_{0}\right) \Rightarrow Q E D \operatorname{cons}\left(s_{2 n}\right) ?
$$

Initial states


## Soundness of SQED

## Theorem: if $\sim Q E D$ cons $\left(s_{2 n}\right)$ then processor $P$ has a bug.

Initial states

$\boldsymbol{i}_{D}$
QED test $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$

## Conditional Completeness of SQED

- Set up and trigger bug.
- $s_{n} \in S_{b}, i_{1}{ }^{\prime}=\operatorname{Dup}\left(i_{1}\right)=i_{b}$.
- Freedom in choosing $L_{D}$.

Initial states

$\boldsymbol{i}_{D}$
Bug-specific QED test $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$

## Conditional Completeness of SQED

- $i_{1}{ }^{\prime}=\operatorname{Dup}\left(i_{1}\right)=i_{b}$ fails in $s_{n}$.
- Goal: bug effect appears in $s_{2 n}$.

Initial states


Setup sequence $\boldsymbol{i}_{0}$
$\boldsymbol{i}_{D}$
Bug-specific QED test $\boldsymbol{i}=i_{O}:: \boldsymbol{i}_{D}$

## Conditional Completeness of SQED

## Theorem: if bug-specific QED test $\boldsymbol{i}$ exists then $\sim Q E D \operatorname{cons}\left(s_{2 n}\right)$.

Initial states

$\boldsymbol{i}_{D}$
Bug-specific QED test $\boldsymbol{i}=\boldsymbol{i}_{O}:: \boldsymbol{i}_{D}$

## Full Completeness of SQED

- No duplication: run twice.
- Run reset instructions.
- Assume: Spec in initial states.

Initial states


QED test $i$ with reset

## Full Completeness of SQED

- No duplication: run twice.
- Run reset instructions.
- Assume: Spec in initial states.

$2^{\text {nd }}$ run

QED test $i$ with reset

## Full Completeness of SQED

Theorem: if P has no failing QED test with reset, then P is correct.

$2^{\text {nd }}$ run

QED test i with reset

## Future Work

Leveraging QED test extensions:

- Soft/hard reset not yet applied in practice.
- Design-for-verification approach.

Formal model refinements:

- Instruction pipelines, multiprocessor systems.
- Deadlock detection.
- Symbolic starting states.


## Thank you for your attention!

