

# DepQBF: A Dependency-Aware QBF Solver (System Description)

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TNF

<i>Solver</i>	<i>Score</i>
<b>DepQBF</b>	<b>2896.68</b>
DepQBF-pre	2508.96
aqme-10	2467.96
qmaiga	2117.55
AIGSolve	2037.22
quantor-3.1	1235.14
struqs-10	947.83
nenofex-qbfeval10	829.11

[http://www.qbflib.org/index\\_eval.php](http://www.qbflib.org/index_eval.php)

### This Talk:

- DepQBF 0.1 system overview.
- Selected features: restarts, removal of learnt constraints.
- Experimental evaluation.

## DepQBF:

- Input: QBFs in Prenex-CNF (PCNF).
- QDLL with conflict-driven clause and solution-driven cube learning.
- Analysis of variable dependencies.

## Variable Dependencies in QBFs:

- PCNF  $Q_1 Q_2 \dots Q_n. \phi$ : linearly ordered sets of quantified variables.
- Left-to-right prefix order: strong dependencies.
- DepQBF: relaxing prefix order by dependency schemes.

## Example

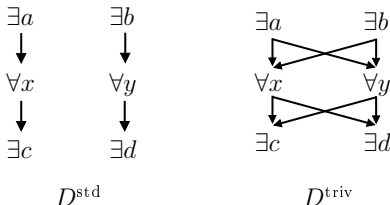
Quantifier ordering matters:

- $\forall x \exists y. (x = y)$  is satisfiable: value of  $y$  *depends* on value of  $x$ .
- $\exists y \forall x. (x = y)$  is unsatisfiable: value of  $y$  is fixed for all values of  $x$ .

**Dependency Schemes:**  $D \subseteq (V_{\exists} \times V_{\forall}) \cup (V_{\forall} \times V_{\exists})$ . [SS09, LB09, LB10, Ben05]

- $(x, y) \notin D$ :  $y$  independent from  $x$ .
- $(x, y) \in D$ : conservatively regard  $y$  as depending on  $x$ .
- DepQBF: *standard dependency scheme*  $D^{\text{std}} \subseteq D^{\text{triv}}$ .
  - Previous work:  $D^{\text{std}}$  as dependency-DAG over equivalence classes.
  - Efficient integration.

**Example:**  $\exists a, b \forall x, y \exists c, d. (a \vee x \vee c) \wedge (a \vee b) \wedge (b \vee d) \wedge (y \vee d)$ .



Standard dependency scheme  $D^{\text{std}}$ , quantifier prefix  $D^{\text{triv}}$ .

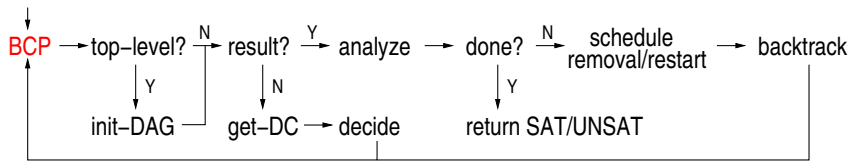


Figure: DepQBF workflow.

## Boolean Constraint Propagation (BCP):

- Propagation of unit and pure literals.
- Watched data-structures for efficient detection.

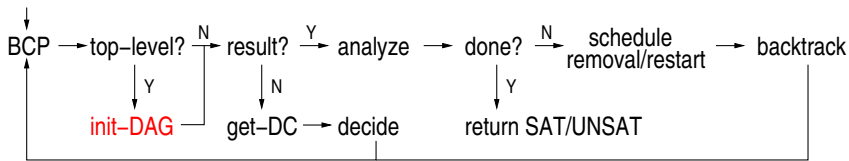


Figure: DepQBF workflow.

### Initialize Dependency-DAG:

- Top-most decision level 0.
- All assignments at top-level are permanent.
- Permanent simplifications (satisfied clauses).
- Potential reduction of dependencies.

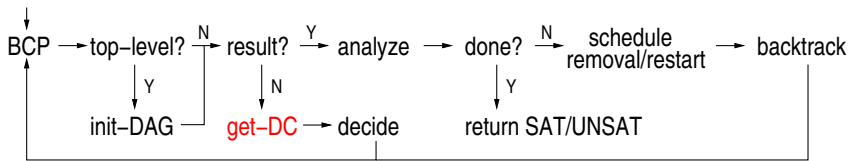


Figure: DepQBF workflow.

## Retrieve Decision Candidates (DC):

- Get possible decision variables (candidates) from dependency-DAG.
- Candidate: all “preconditions” (predecessors in DAG) assigned.
- Candidate set is maintained incrementally and lazily.

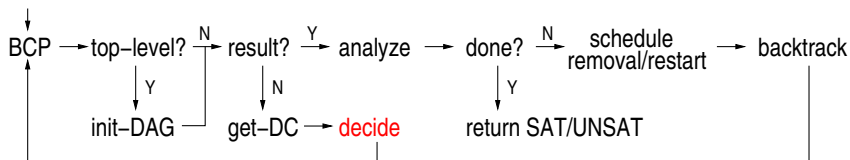


Figure: DepQBF workflow.

### Decision Making:

- Select decision variable from candidate set.
- Activity-based priority queue of variables (VSIDS, like MiniSAT 2).
- Assignment caching.



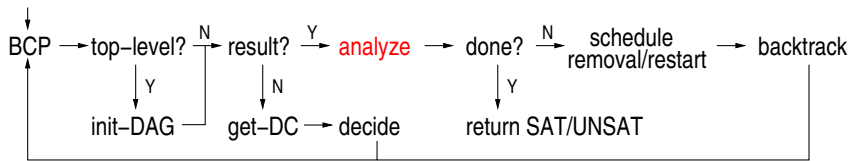


Figure: DepQBF workflow.

### Constraint Learning (Result Analysis):

- Conflict/solution: generate *asserting* learnt clause/cube.
- Augmented CNF:  $\phi := \phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$ .
- Learnt clauses  $\phi_{LCL}$  and cubes  $\phi_{LCU}$ .
- Q-resolution/consensus to derive learnt clauses/cubes.
- See also our SAT'10 paper.

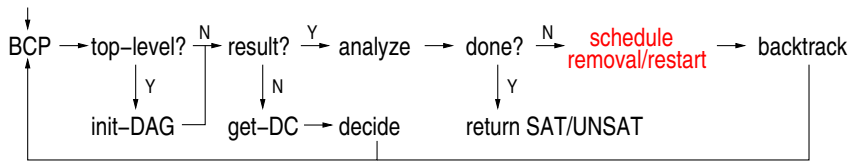


Figure: DepQBF workflow.

## Learnt Constraint Removal and Restarts:

- Check each time when adding a new learnt constraint.
- Capacity exhausted: remove half of learnt constraints.
- Heuristically try to keep “useful” constraints, increase capacity.
- Inner-outer restart schedule (like PicoSAT).

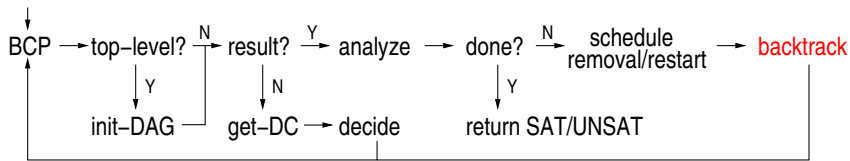


Figure: DepQBF workflow.

## Backtracking:

- General (frequent) case: backtrack to asserting level of learnt constraint.
- Special case: backtrack to restart level.

**Learnt Constraints:** [GNT02, Let02, ZM02, GNT06, BKF95, GS08, ES03, GN02]

- Clauses  $\phi_{LCL}$  and cubes  $\phi_{LCU}$ , stored in doubly-linked lists.
- Initial capacities depend on formula size: [2500, 10000].

**Move-To-Front (MTF) Strategy:** approximating clause activities.

- Want to keep “used” (i.e. important?) constraints: units, learning.
- Move used constraints  $C_i$  to head of list:

$$\{ \underbrace{C_1, \dots, C_{i-1}}_{\text{most-recently used}}, \underbrace{C_i}_{\text{least-recently used}}, C_{i+1}, \dots, C_n \} \xrightarrow{MTF(C_i)} \{ \underbrace{C_i}_{\text{deletion order}}, C_1, \dots, C_{i-1}, C_{i+1}, \dots, C_n \}$$

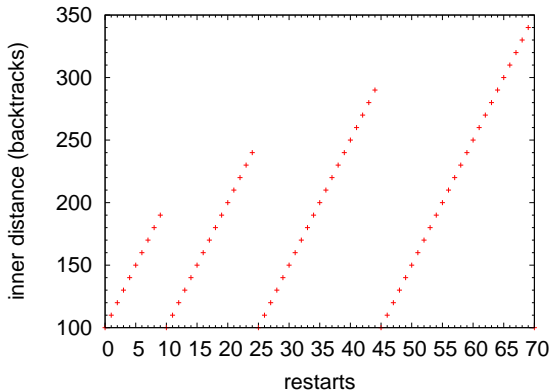
**Deletion:**

- Capacity exhausted: remove half of constraints, starting at tail of list.
- Least-recently used ones are deleted (hopefully: least-important ones).
- Increase capacity by constant 500.

**Inner-Outer Restart Schedule:** when to restart?

[Bie08]

- Inspired by PicoSAT: separate inner/outer restarts.
- Inner restart after  $i$  backtracks, outer restart after  $o$  inner restarts.
- Initially  $i := 100$ ,  $o := 10$ .
- Before  $i$ th ordinary backtrack: jump to *restart level* instead,  $i := i + 10$ .
- After  $o$  inner restarts:  $i := 100$ ,  $o := o + 5$  (outer restart).

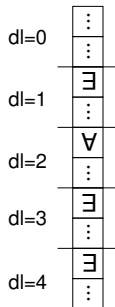


**Restart Level:** where to jump to?

- Normally, DepQBF always jumps to asserting level.
- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**

- Assignment stack, in order of decision levels.
- Conflict/solution at level 4.
- Restart is scheduled, where to jump to?

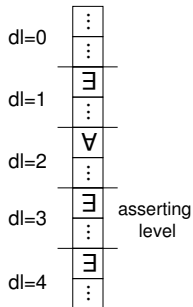


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**Example:**

- Current learnt constraint asserting at level 3.
- Last universal decision at level 2.

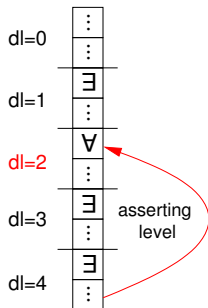


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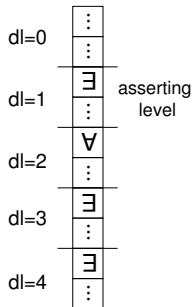


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- Restart: possibly jump *most-recent universal decision level* instead.
  - Always the longer jump is taken.
- Related to ideas from unrestricted backtracking [BLdSMS05].

**Example:**

- Current learnt constraint asserting at level 1.
- Last universal decision at level 2.

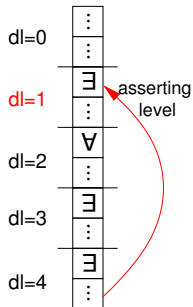


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- Normally, DepQBF always jumps to asserting level.
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**Example:**

- Current learnt constraint asserting at level 1.
- Last universal decision at level 2.
- **Restart: take the longer jump.**



	<i>All</i>		<i>Solved SAT</i>		<i>Solved UNSAT</i>	
	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>
<i>without preprocessing</i>						
DepQBF	370	337.10	165	54.58	205	20.82
DepQBF-nr	360	352.33	154	51.36	206	24.35
DepQBF-nc	350	384.66	157	107.48	193	28.05
DepQBF-np	345	398.12	141	114.72	204	45.37
DepQBF-ncnr	340	400.24	147	124.10	193	20.19
QuBE7.0-nopp	332	425.44	135	147.71	197	47.27
QuBE6.6-nopp	301	468.51	113	136.48	188	55.27

**Table:** QBFEVAL'10 main track (568 formulae). Ranking by number of solved formulae.

### Setup:

- Ubuntu 9.04, Intel® Q9550@2.83 GHz, 3 GB/900 sec.
- DepQBF: version 0.1 which participated in QBFEVAL'10.

	<i>All</i>		<i>Solved SAT</i>		<i>Solved UNSAT</i>	
	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>
<i>without preprocessing</i>						
<b>DepQBF</b>	<b>370</b>	<b>337.10</b>	<b>165</b>	<b>54.58</b>	<b>205</b>	<b>20.82</b>
<b>DepQBF-nr</b>	<b>360</b>	<b>352.33</b>	<b>154</b>	<b>51.36</b>	<b>206</b>	<b>24.35</b>
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### Important:

- Restarts (disabled in DepQBF-nr).

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### Important:

- Restarts.
- Assignment caching.
- Pure literal detection (disabled in DepQBF-np).

	<i>All</i>		<i>Solved SAT</i>		<i>Solved UNSAT</i>	
	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>	<i>solved</i>	<i>avg.time</i>
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### Important:

- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching (disabled in DepQBF-ncnr).

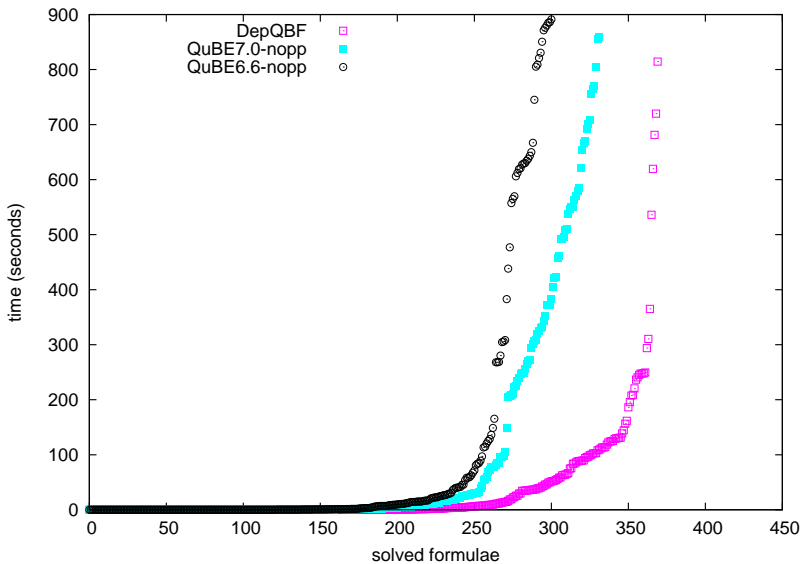
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<b>QuBE7.0-pre<math>\rightarrow</math>DepQBF</b>	<b>424</b>	<b>254.23</b>	<b>197</b>	<b>48.17</b>	<b>227</b>	<b>23.42</b>
<b>QuBE7</b>	<b>414</b>	<b>310.29</b>	<b>187</b>	<b>130.52</b>	<b>227</b>	<b>58.33</b>
<b>QuBE6.6</b>	<b>387</b>	<b>341.91</b>	<b>168</b>	<b>98.97</b>	<b>219</b>	<b>67.03</b>
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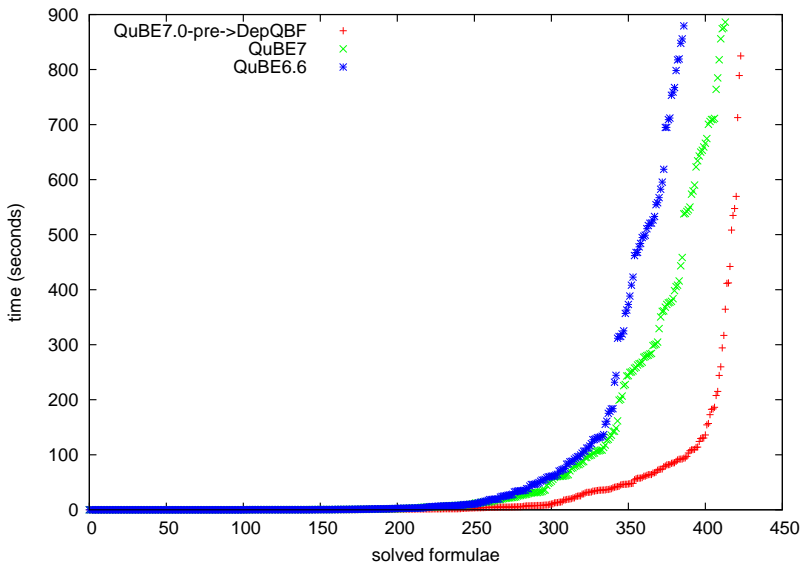
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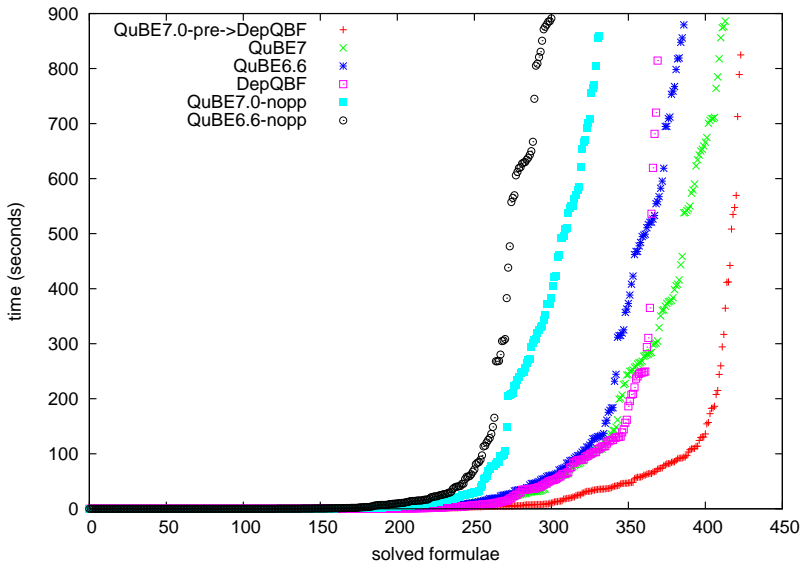
- Restarts.
- Assignment caching.
- Pure literal detection.
- Combining restarts with assignment caching.
- Preprocessing (**not** part of DepQBF 0.1, disabled in QuBE\*-nopp).







# Experiments: QBFEVAL'10 Main Track



## DepQBF:

[BLB10]

- Search-based QBF solver with clause- and cube-learning.
- Relaxing prefix order by dependency-DAG for  $D^{\text{std}}$ .
- Approaches from SAT domain.
- Development:
  - Fuzz testing using QBFuzz: <http://fmv.jku.at/qbfuzz/>
  - Delta-debugging using QBFDD: <http://fmv.jku.at/qbfdd/>
  - Cross-checking against other solvers, mainly QuBE.

## Performance:

- Top-ranked solver in QBFEVAL'10.
- DepQBF 0.1 does *not* include preprocessing.
- But: preprocessing is *very* important.

## Future Work:

- Preprocessing, parameter tuning, decision heuristics, ...

DepQBF 0.1 is open source: <http://fmv.jku.at/depqbf/>

**Unit Clauses:** Clause  $C$  is unit iff

[CGS98, GGN<sup>+</sup>03, MMZ<sup>+</sup>01, GNT07]

- no  $l \in C$  is true.
- exactly one  $l_e \in L_{\exists}(C)$  is unassigned.
- for all unassigned  $l_u \in L_{\forall}(C)$ :  $l_u \not\prec l_e$ , i.e.  $\text{Var}(l_u), \text{Var}(l_e)$  independent.
- Dependency checking  $\prec$  with respect to dependency scheme.
- Dual definition for cubes.

**Two-Literal-Watching:**

- Watch two unassigned literals  $l_1, l_2 \in C$  such that
  - (1) either  $q(l_1) = q(l_2) = \exists$ , or
  - (2)  $q(l_1) = \forall, q(l_2) = \exists$  and  $l_1 \prec l_2$ .

**Watcher Update:**

- Dependency checking needed only in case (2).
- Stop when finding satisfying literal.
- No work needed during backtracking.

**Pure Literals (PL):**[CGS98, GGN<sup>+</sup>03, GNT04]

- Variable has only positive/negative literals left.
- Assigning  $\forall$ -PLs/ $\exists$ -PLs can trigger new units/further PLs.
- Drawback: expensive detection in  $\phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$ .

**Spurious Pure Literals (SPL):**

- Def.: Variable is pure (SPL) if it is pure in original clauses  $\phi_{OCL}$  only.
- SPL-Detection neglects all learnt constraints in  $(\phi_{LCL} \vee \phi_{LCU})$ .
  - Advantage: more efficient detection.
- Variable might be pure in  $\phi_{OCL}$  but not in  $\phi_{OCL} \wedge (\phi_{LCL} \vee \phi_{LCU})$ .
  - Drawback: must ignore such SPL-implications in  $(\phi_{LCL} \vee \phi_{LCU})$ .

**Clause Watching:**

- Positive/negative occurrences  $C(x), C(\bar{x}) \subseteq \phi_{OCL}$ .
- Watch two unsatisfied clauses  $C_x \in C(x)$  and  $C_{\bar{x}} \in C(\bar{x})$ .

**Clause Watcher Update:**

- Assign  $x/\bar{x}$ : all clauses in  $C(x)/C(\bar{x})$  will be satisfied.
- Update watchers of variables  $y$  watching clauses in  $C(x)/C(\bar{x})$ .

**Notification Lists:**

- Goal: avoid searching for variables which need watcher update.
- Lists  $NL_x/NL_{\bar{x}}$  of variables  $y$  watching clauses in  $C(x)/C(\bar{x})$ .
- Assign  $x/\bar{x}$ :
  - *exactly* all variables in  $NL_x/NL_{\bar{x}}$  must update their watcher.
  - update  $NL_x/NL_{\bar{x}}$  of variables  $x$  occurring in old and new watched clauses.
- No work needed during backtracking.

## Activity-Based Variable Priority Queue:

[MMZ<sup>+</sup>01, ES03]

- DepQBF: straight-forward generalization of idea from SAT domain.
- Maintain VSIDS score (activity) for each variable.
- Increase activity of variables encountered during learning.
- Periodically down-scale activities.
- Implementation follows MiniSAT 2.
- Decision making: select *candidate* with highest activity.
- Lazy priority queue maintenance (like in MiniSAT):
  - Discard assigned variables and non-candidates on the fly upon removal.



### Also called: Phase Saving

[PD07]

- DepQBF: straight-forward generalization of idea from SAT-domain.
- Each variable has a cached assignment (possibly undefined).
- All assignments (unit, pure literals, decisions) update cache.
- Decision variables: assign cached value, if any.
- No distinction between different quantifiers.

<i>Suite mqm (136 formulae)</i>		
	<i>solved</i>	<i>avg.time</i>
DepQBF	136	39.83
QuBE7	117	306.43
QuBE7.0-nopp	115	304.82
QuBE6.6	100	393.93
QuBE6.6-nopp	97	399.55

Table: Solvers sorted by number of solved formulae.

### Benchmark Suite *mqm*:

- Minimal Query Inseparability Module Extraction in DL-Lite.
- Newly submitted to QBFEVAL'10 by Roman Kontchakov.
- As the only solver, DepQBF solved entire suite in QBFEVAL'10.

QBFEVAL'10: solved formulae only						
	$\cap$		SAT- $\cap$		UNSAT- $\cap$	
<i>solved</i>	328		132		196	
<i>avg.time</i>	84.97	<b>21.87</b>	140.16	<b>32.43</b>	47.81	<b>14.75</b>
QBFEVAL'10: unique results						
	$\Leftrightarrow$		SAT- $\Leftrightarrow$		UNSAT- $\Leftrightarrow$	
<i>solved</i>	<b>86</b>	42	<b>55</b>	33	<b>31</b>	9

Table: QuBE7 (left columns) vs. DepQBF (right columns).

QBFEVAL'10: solved formulae only						
	$\cap$		SAT- $\cap$		UNSAT- $\cap$	
<i>solved</i>	308		115		193	
<i>avg.time</i>	80.14	<b>17.49</b>	114.17	<b>23.23</b>	59.86	<b>14.07</b>
QBFEVAL'10: unique results						
	$\Leftrightarrow$		SAT- $\Leftrightarrow$		UNSAT- $\Leftrightarrow$	
<i>solved</i>	<b>79</b>	62	<b>53</b>	50	<b>26</b>	12

Table: QuBE6.6 (left columns) vs. DepQBF (right columns).



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