

# A Theoretical Framework for Symbolic Quick Error Detection

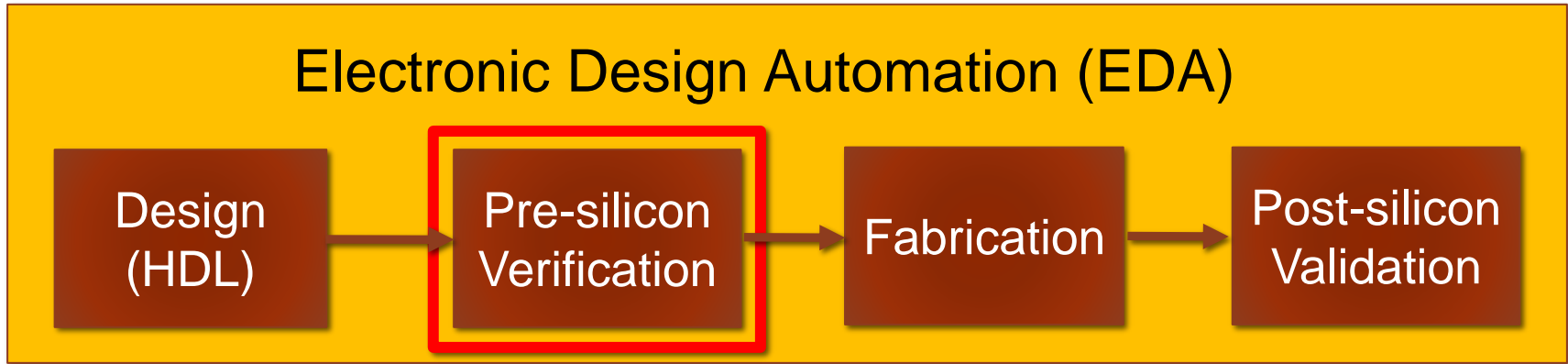


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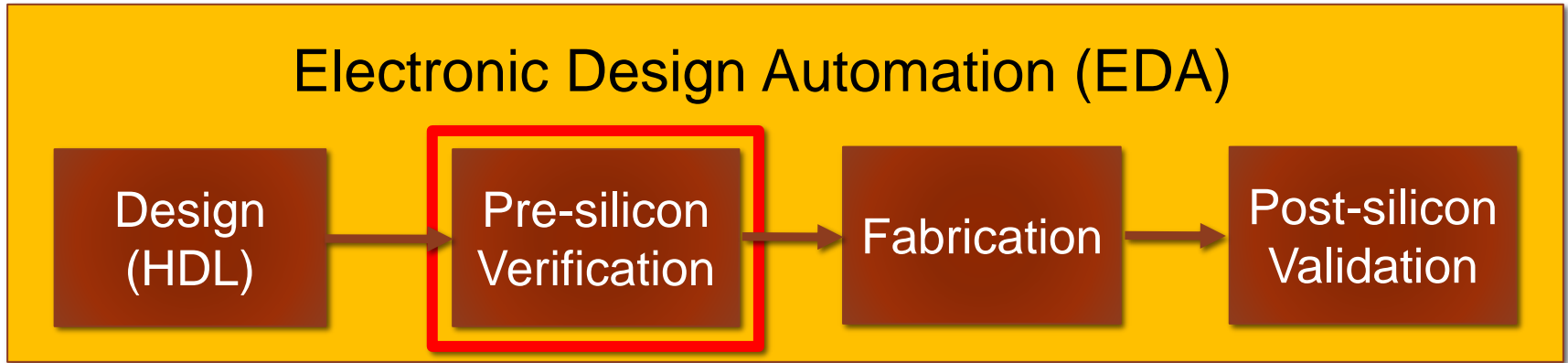
Preprint: <https://arxiv.org/abs/2006.05449>

# Context: Pre-Silicon Verification



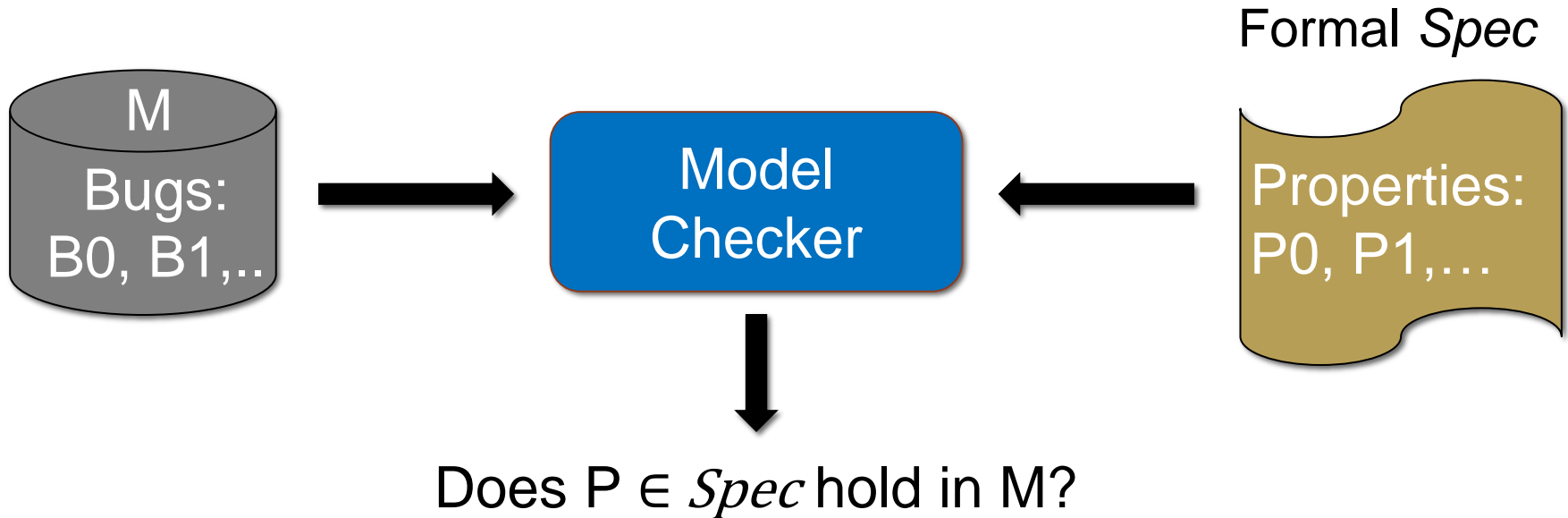
- Our focus: processor designs.
- Formally verify model of a design (e.g. Verilog).
- Model checking vs. non-formal simulation or testing.

# Context: Pre-Silicon Verification



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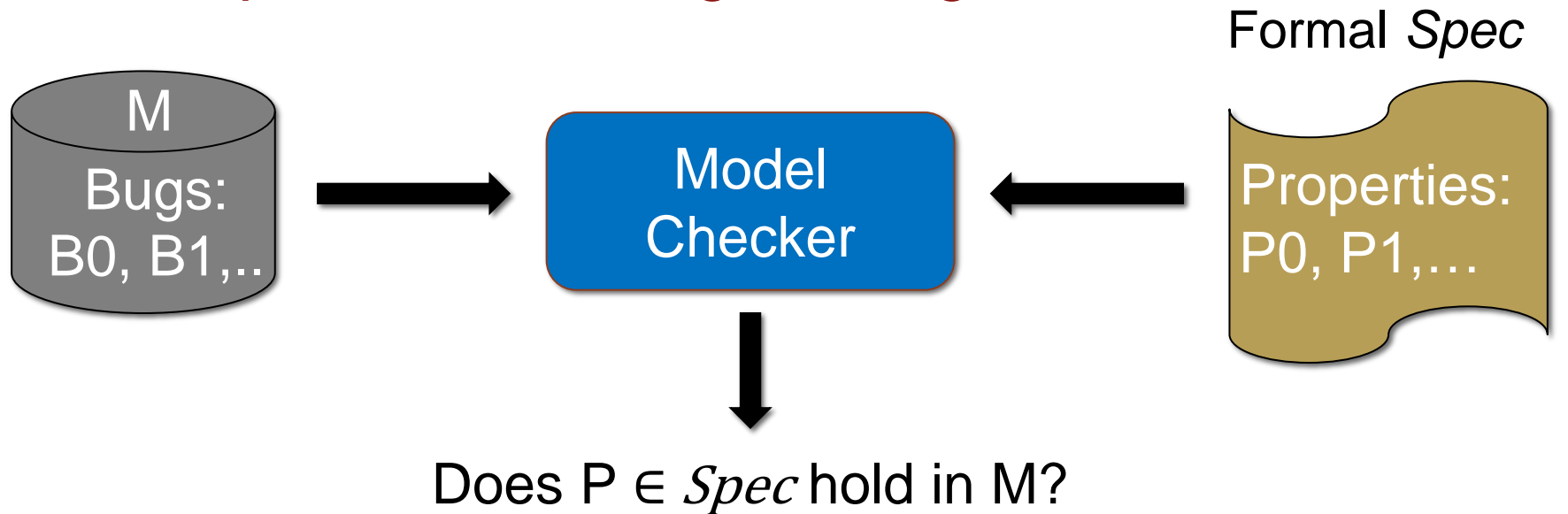
# Soundness of Bug-Finding



- If  $P \in \textit{Spec}$  fails then  $B \in M$ .
- Property  $P$  covers bug  $B$ .

Soundness  $\approx$   
no spurious cex

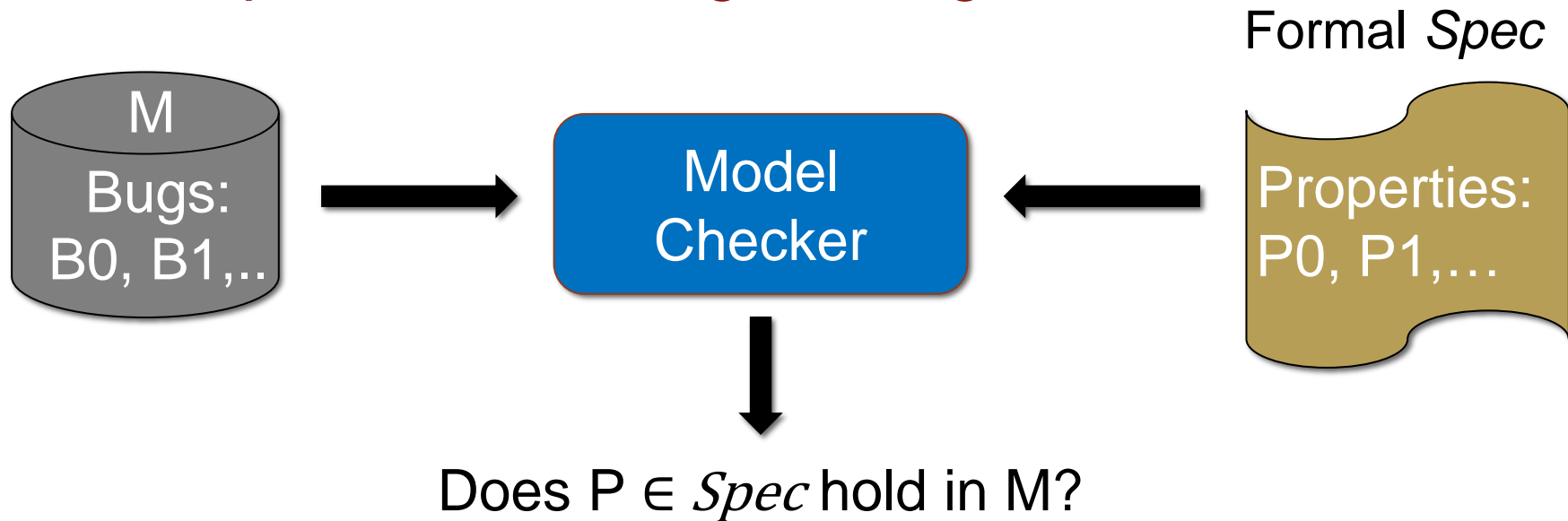
# Completeness of Bug-Finding



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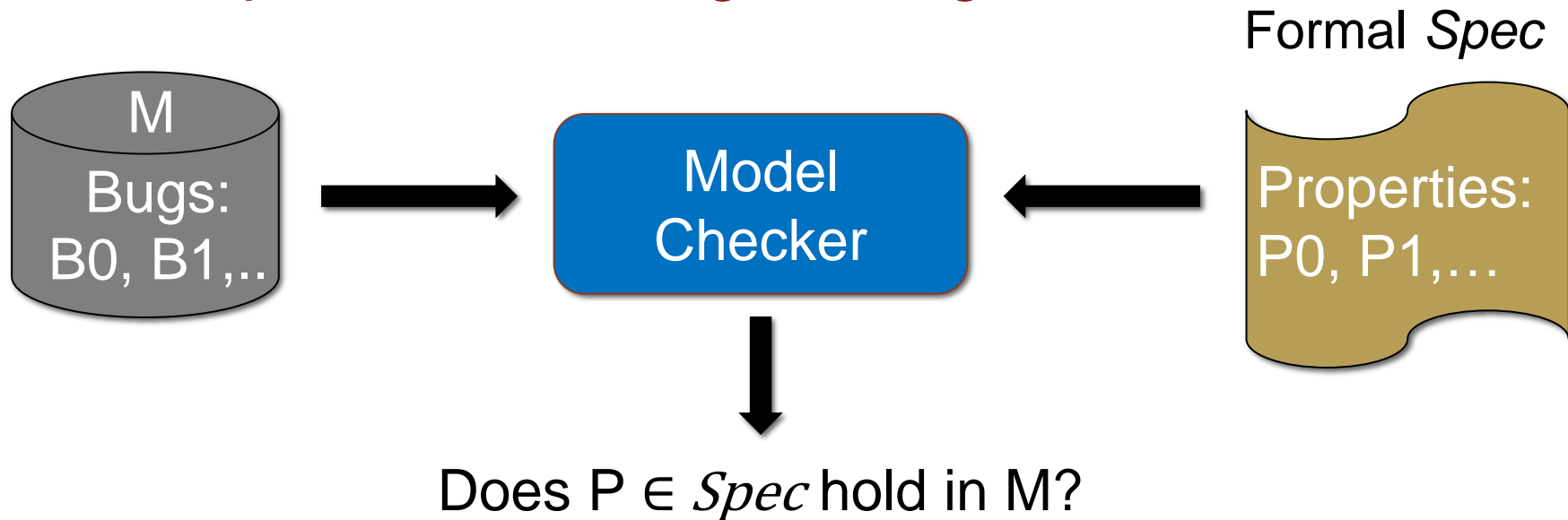
Completeness  $\approx$   
*Spec* covers all bugs

# Completeness of Bug-Finding



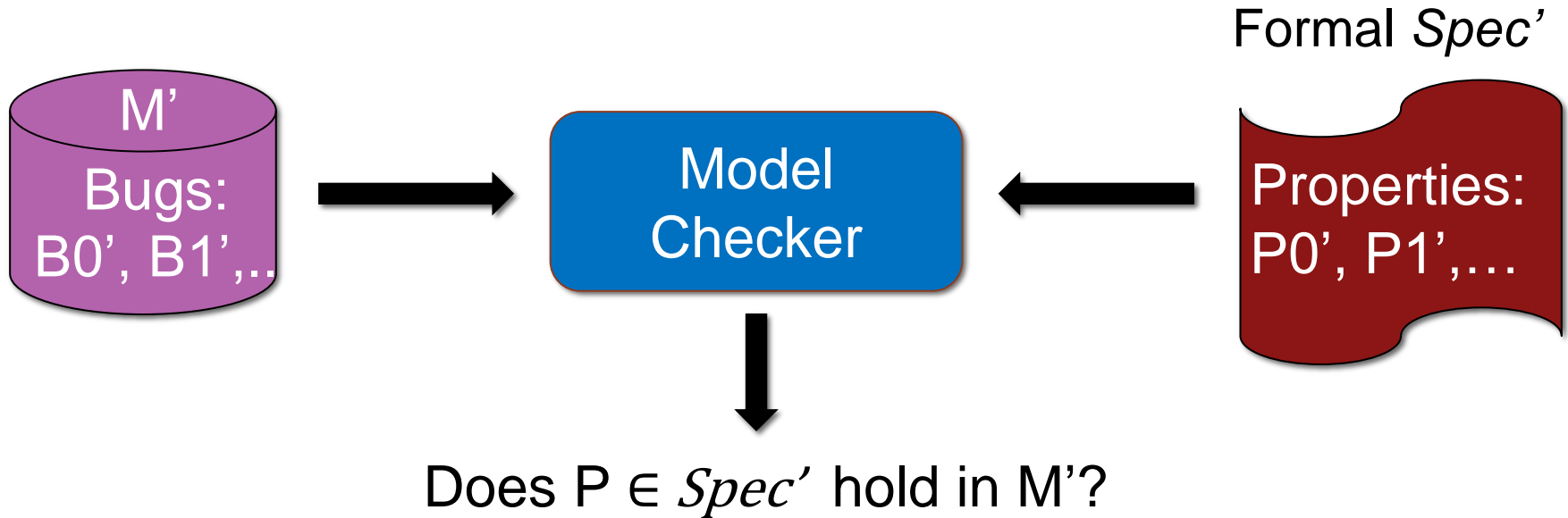
- “Have I written enough properties?” [Katz et al. CHARME’99].
- **Challenge:** making *Spec* complete.

# Completeness of Bug-Finding



- *Spec*: manual writing of implementation-specific properties.

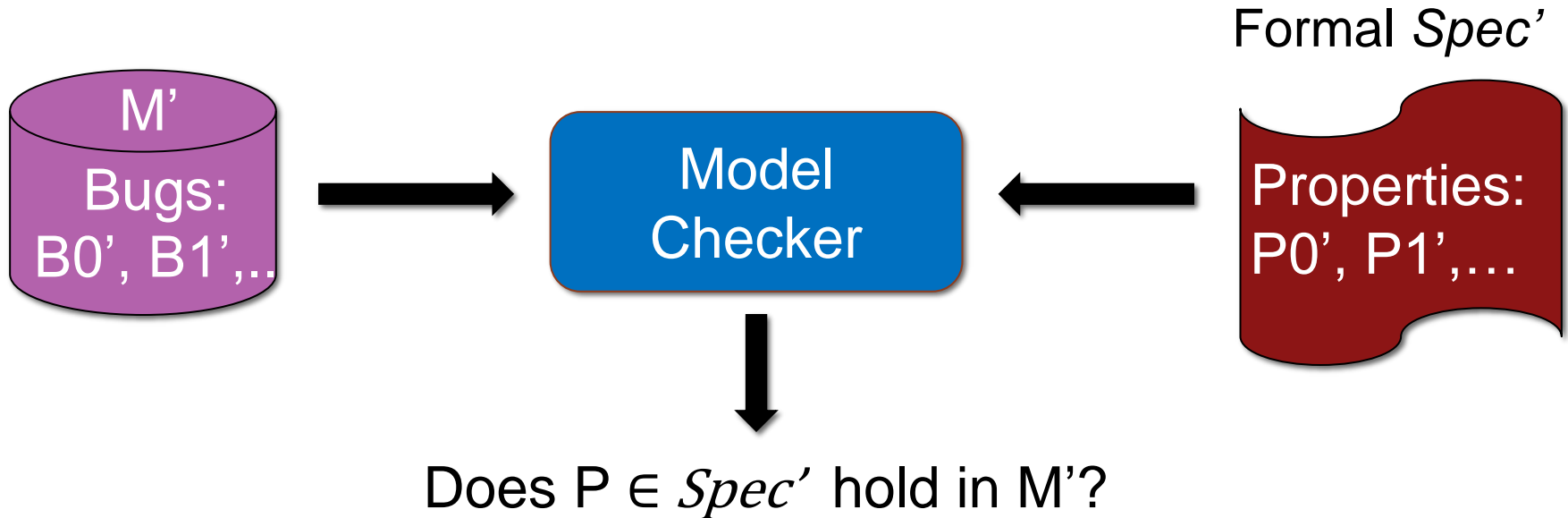
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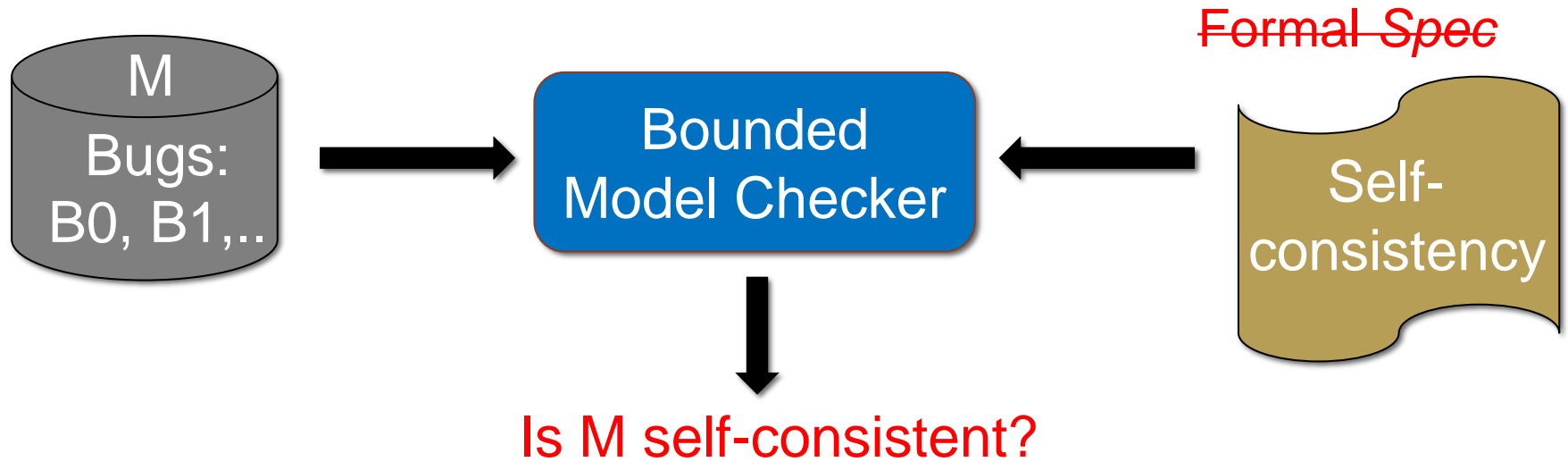


# Completeness of Bug-Finding



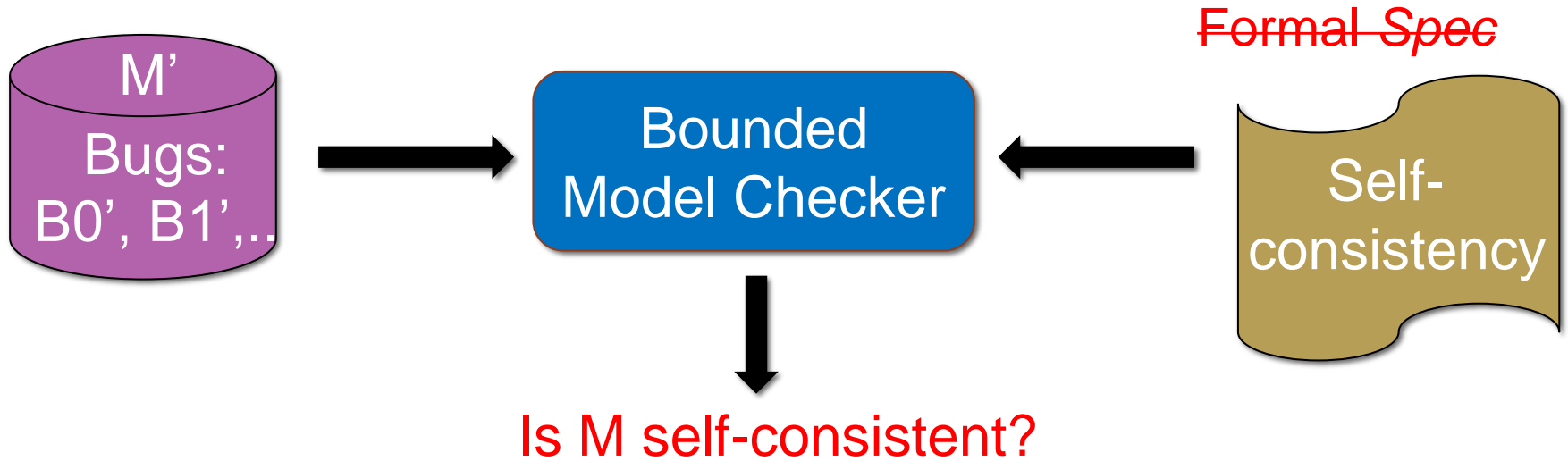
- *Spec*: manual writing of implementation-specific properties.
- Model/design changes  $\rightarrow$  *Spec* to be adapted (manually).
- Completeness depending on *Spec*.

# Symbolic Quick Error Detection (SQED)



- **No** need for *Spec* or implementation-specific properties.
- Leverages bounded model checking (BMC).

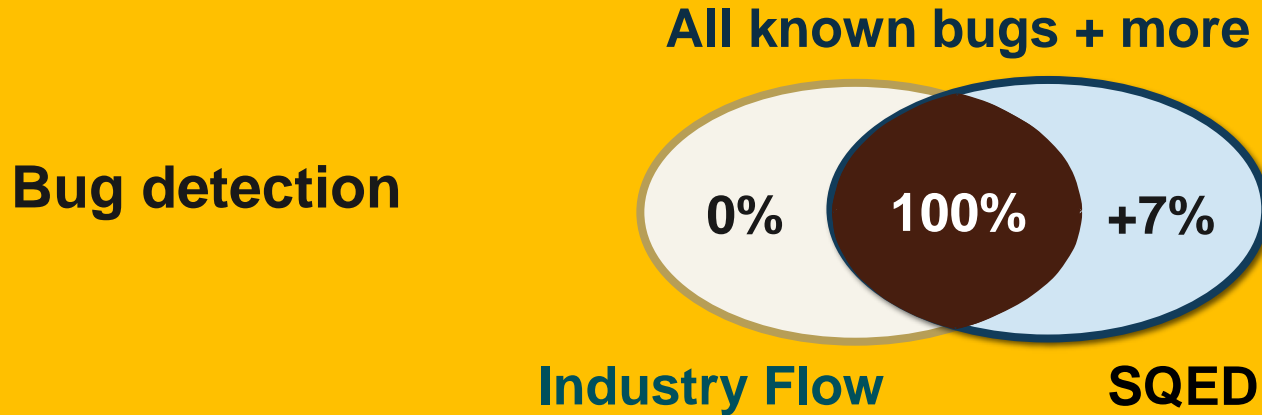
# Symbolic Quick Error Detection (SQED)



- **No** need for *Spec* or implementation-specific properties.
- Leverages bounded model checking (BMC).
- **Self-consistency**: universal property, **no** manual writing.

# SQED: Industrial Strength

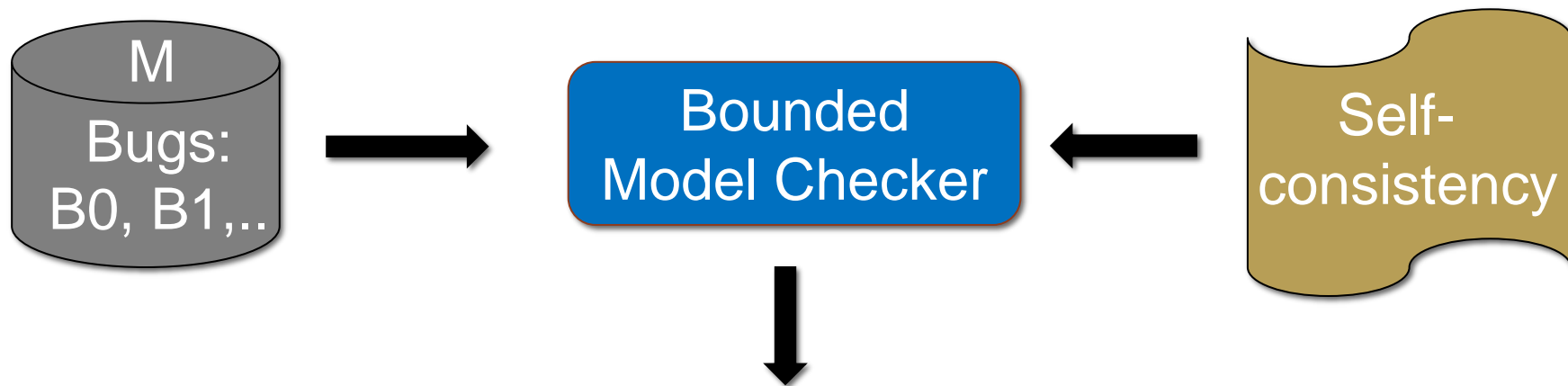
INFINEON case study: automotive IP versions [Singh et al DATE'19]



**Traditional verification:**

(Constrained) random simulation, directed tests, formal.

# Our Contributions: SQED Formal Proofs



Does  $P \in Spec$  hold in  $M$ ?  $\approx$  Is  $M$  self-consistent?

1. Soundness: no spurious cex.
2. (Conditional) completeness: all bugs covered (BMC depth).
3. Formal framework: abstract processor model.

# Self-Consistency

- Function  $f$ : equivalent inputs  $\rightarrow$  equivalent outputs.
- Functional congruence property:

$$\forall x, x' : x = x' \rightarrow f(x) = f(x')$$

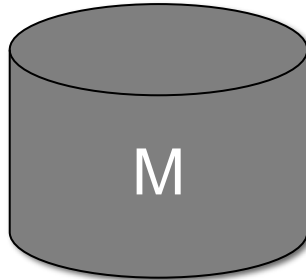
# Self-Consistency

- Processor Design M:

$i_1, i_2, \dots, i_n +$   
inputs (regs/mem)

**=**

$i_1', i_2', \dots, i_n' +$   
inputs' (regs/mem)



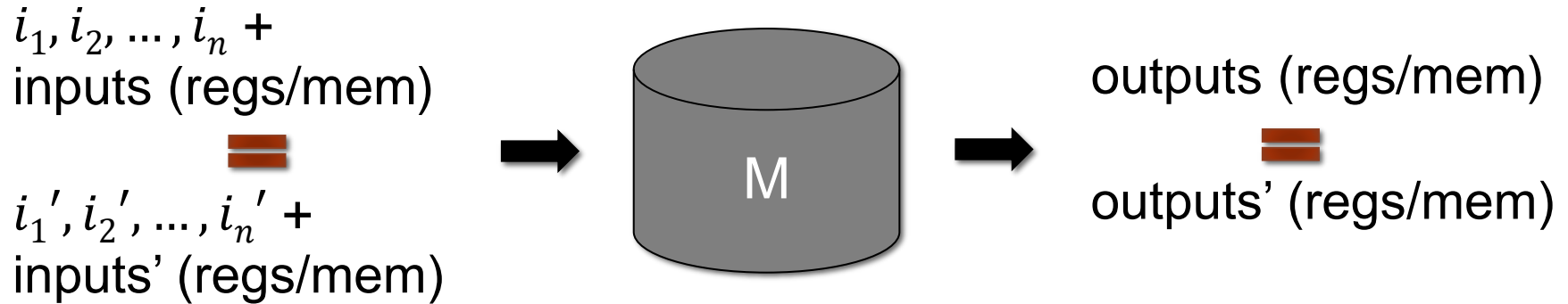
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# Self-Consistency

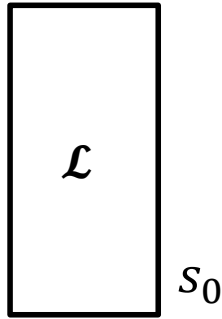
- Processor Design M:



HW designs have complex internal state (pipeline,...).

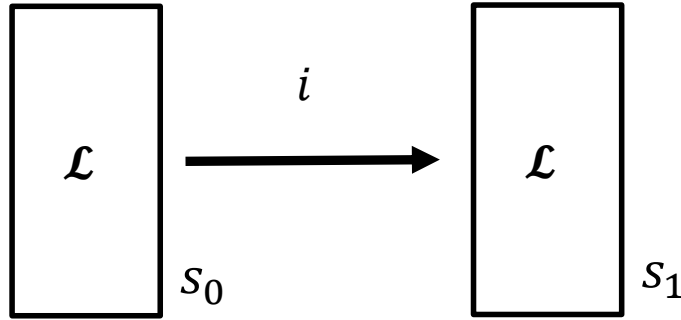


# Formal Model of Processors and SQED



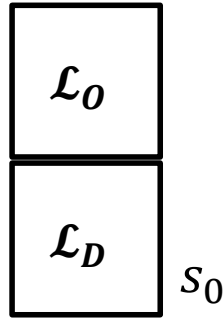
- State  $s_0$ : mapping from locations  $\mathcal{L}$  to values.
- (Non-)architectural parts of  $s_0 = (s_a, s_{na})$ .
- $\mathcal{L}$ : regs. and mem. locations, value  $s_0(l) = s_a(l) = v$ .

# Formal Model of Processors and SQED



- Instruction  $i = (op, l, (l', l''))$ , one-step execution.
- Opcode  $op$ , input locations  $(l', l'')$ , output location  $l$ .
- Transition:  $T(s_0, i) = s_1$ ,  $s_0 = (s_a, s_{na})$ ,  $s_1 = (s_a', s_{na}')$ .

# Formal Model of Processors and SQED



Example: register identifiers

$$\mathcal{L} = \{0, 1, \dots, 31\}$$

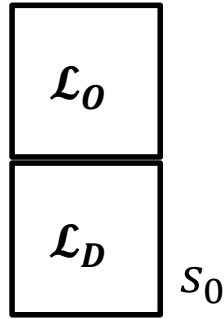
$$\mathcal{L}_O = \{0, \dots, 15\}$$

$$\mathcal{L}_D = \{16, \dots, 31\}$$

$$L_D(l) = l + 16$$

- Partition of  $\mathcal{L}$ : original and duplicate locations  $\mathcal{L}_O, \mathcal{L}_D$ .
- Arbitrary, fixed bijective mapping  $L_D : \mathcal{L}_O \rightarrow \mathcal{L}_D$ .
- **Self-consistency property** based on mapping  $L_D$ .

# Formal Model of Processors and SQED



Example:

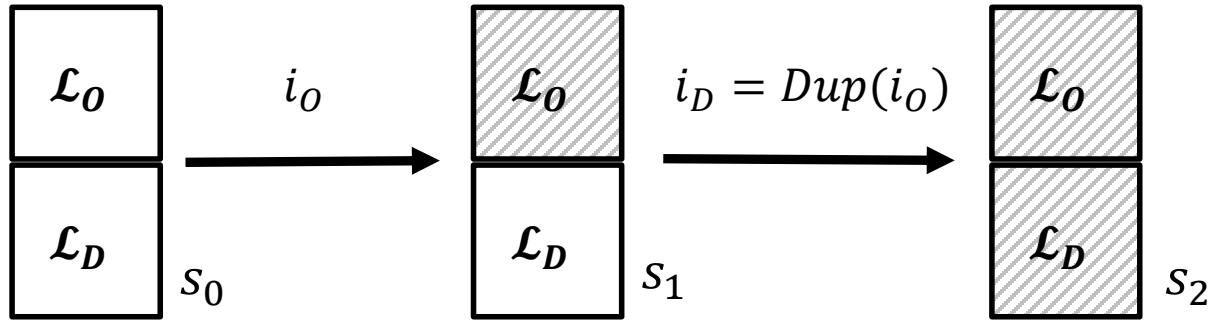
$$i_O = (\text{ADD}, l_{12}, (l_4, l_8))$$

$$L_D(l) = l + 16$$

$$i_D = (\text{ADD}, l_{28}, (l_{20}, l_{24}))$$

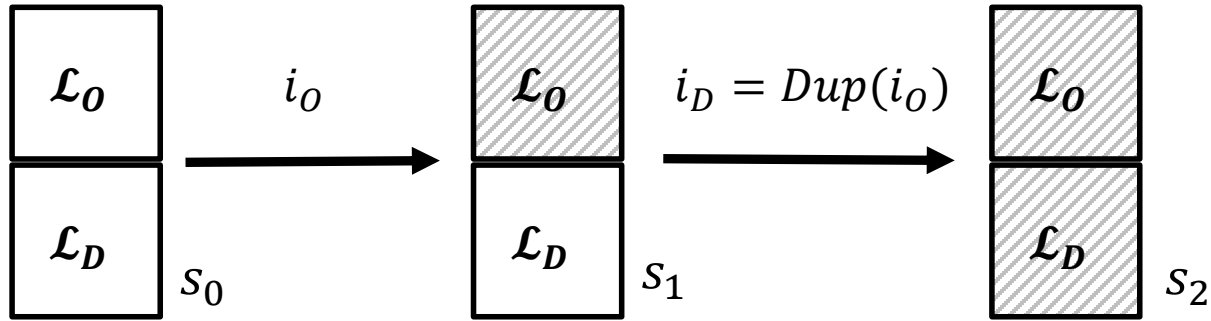
- Original instruction  $i_O = (op, l, (l', l''))$ .
- Duplicate  $i_D = Dup(i_O) = (op, L_D(l), L_D(l', l''))$ .

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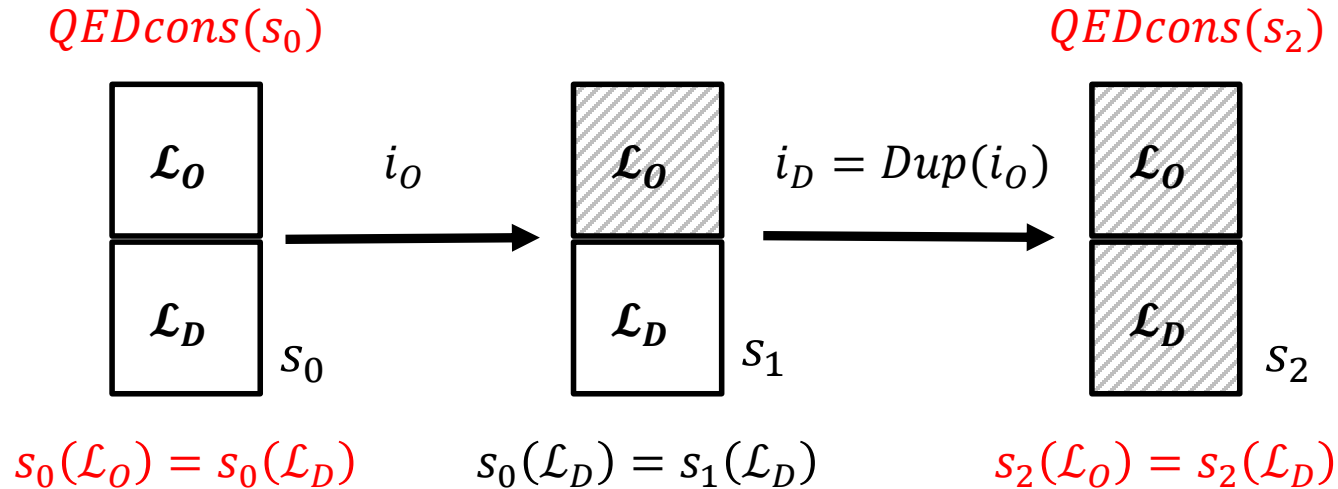
- Original instruction  $i_O = (op, l, (l', l''))$ .
- Duplicate  $i_D = Dup(i_O) = (op, L_D(l), L_D(l', l''))$ .
- Original/duplicate  $i_O/i_D$  operates on  $\mathcal{L}_O/\mathcal{L}_D$  only.

# Formal Model of Processors and SQED



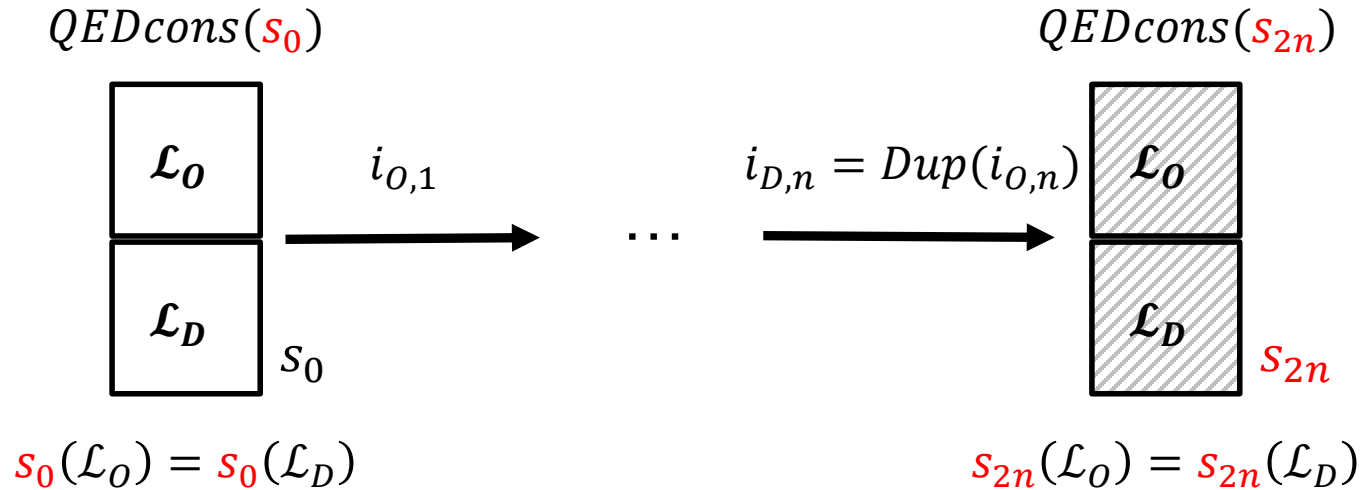
- Given  $\mathcal{L}_D$ , state  $s_0$  **QED-consistent**  $\leftrightarrow s_0(\mathcal{L}_O) = s_0(\mathcal{L}_D)$ .
- Matching values at original/duplicate locations.

# Formal Model of Processors and SQED



- Given  $\mathcal{L}_D$ , state  $s_0$  **QED-consistent**  $\leftrightarrow s_0(\mathcal{L}_O) = s_0(\mathcal{L}_D)$ .
- Matching values at original/duplicate locations.
- **Correct** execution of  $i_0/i_D$  preserves QED-consistency.

# Formal Model of Processors and SQED



- $\mathbf{i}_O = i_{O,1}, \dots, i_{O,n}$  and  $\mathbf{i}_D = i_{D,1}, \dots, i_{D,n}$  with  $\mathbf{i}_D = Dup(\mathbf{i}_O)$ .
- **QED test:** concatenation  $\mathbf{i} = \mathbf{i}_O :: \mathbf{i}_D$  of  $2n$  instructions.
- **Correct** execution of  $\mathbf{i}$  preserves QED-consistency.

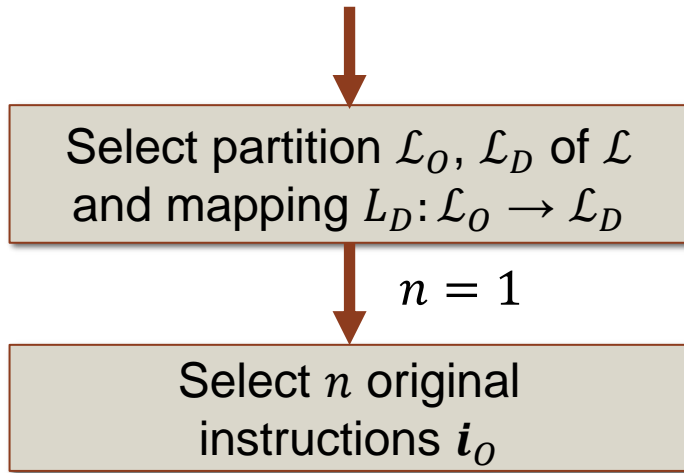


## Using BMC in SQED

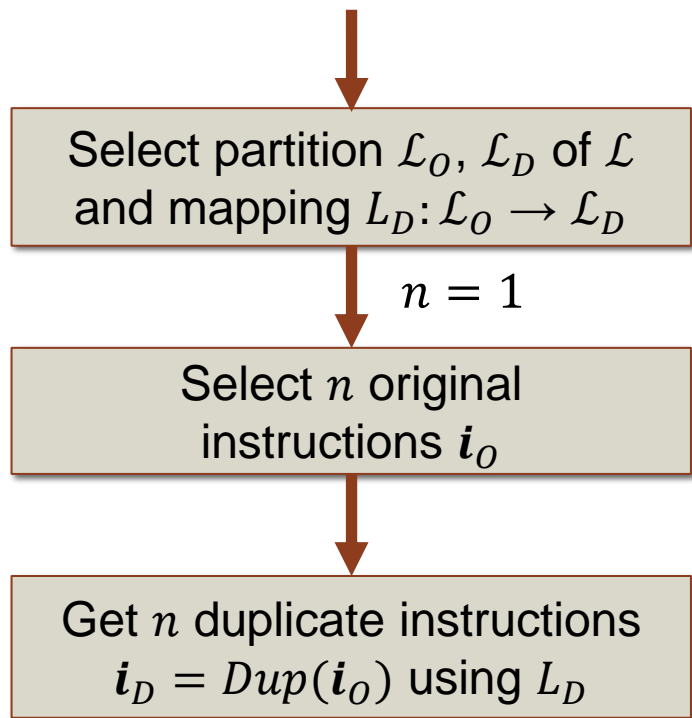


Select partition  $\mathcal{L}_O, \mathcal{L}_D$  of  $\mathcal{L}$   
and mapping  $L_D: \mathcal{L}_O \rightarrow \mathcal{L}_D$

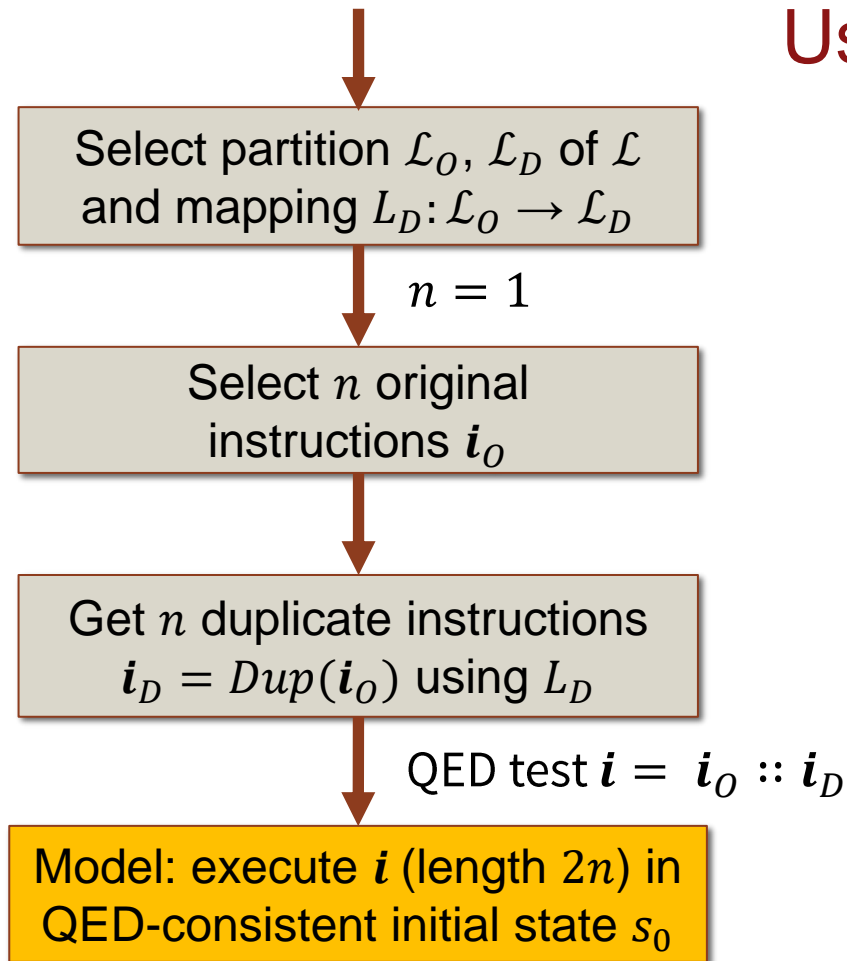
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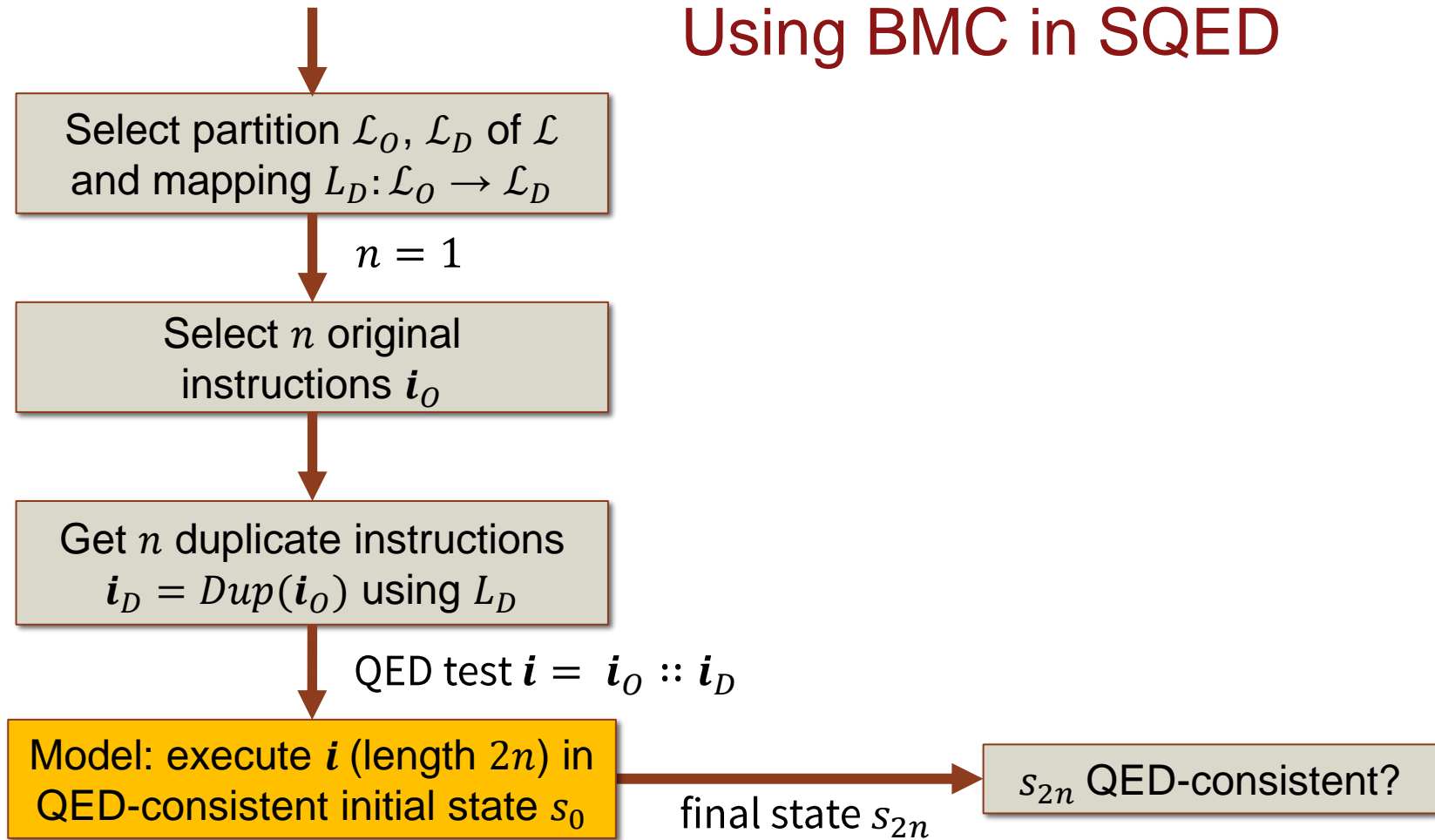
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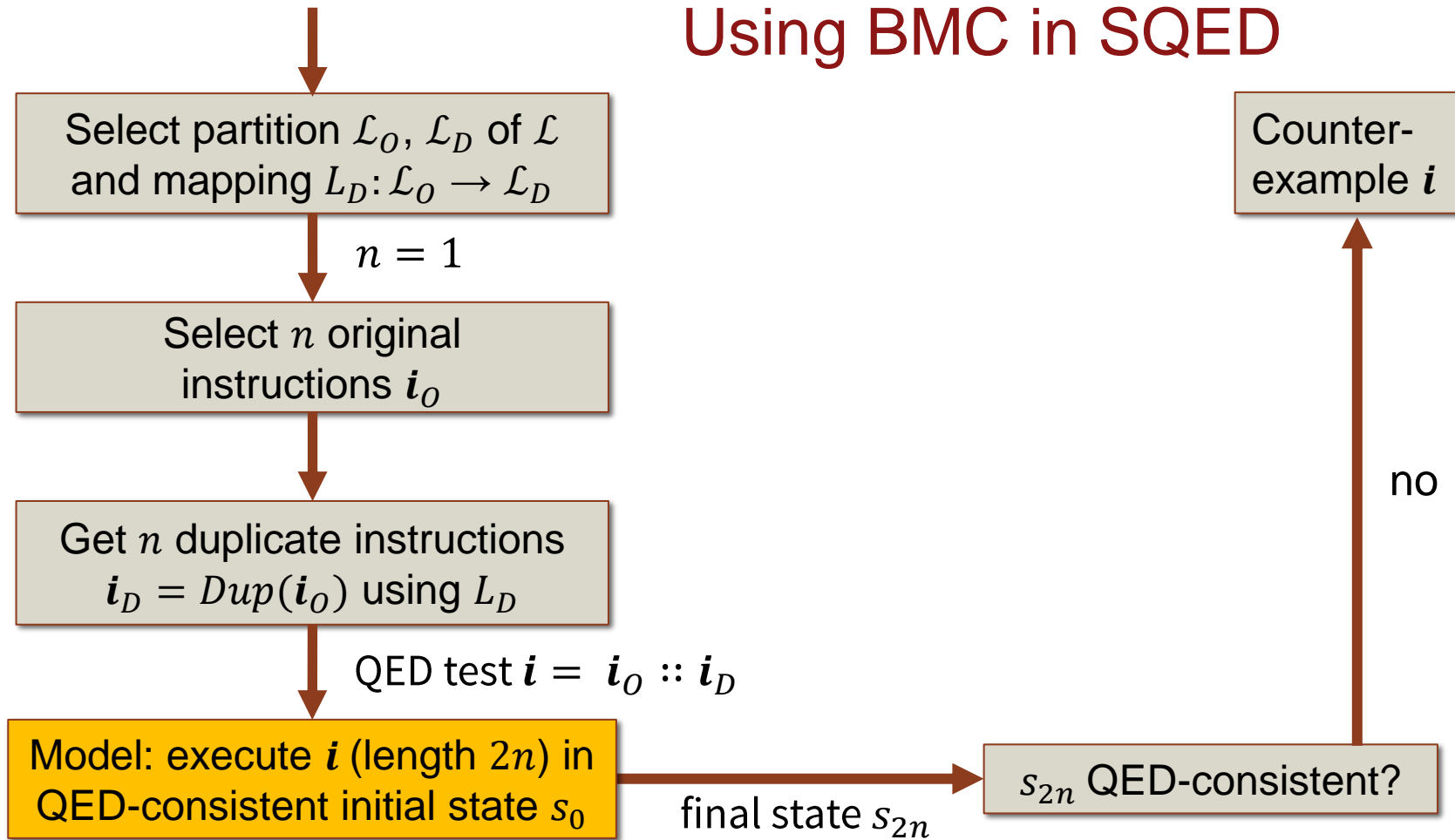
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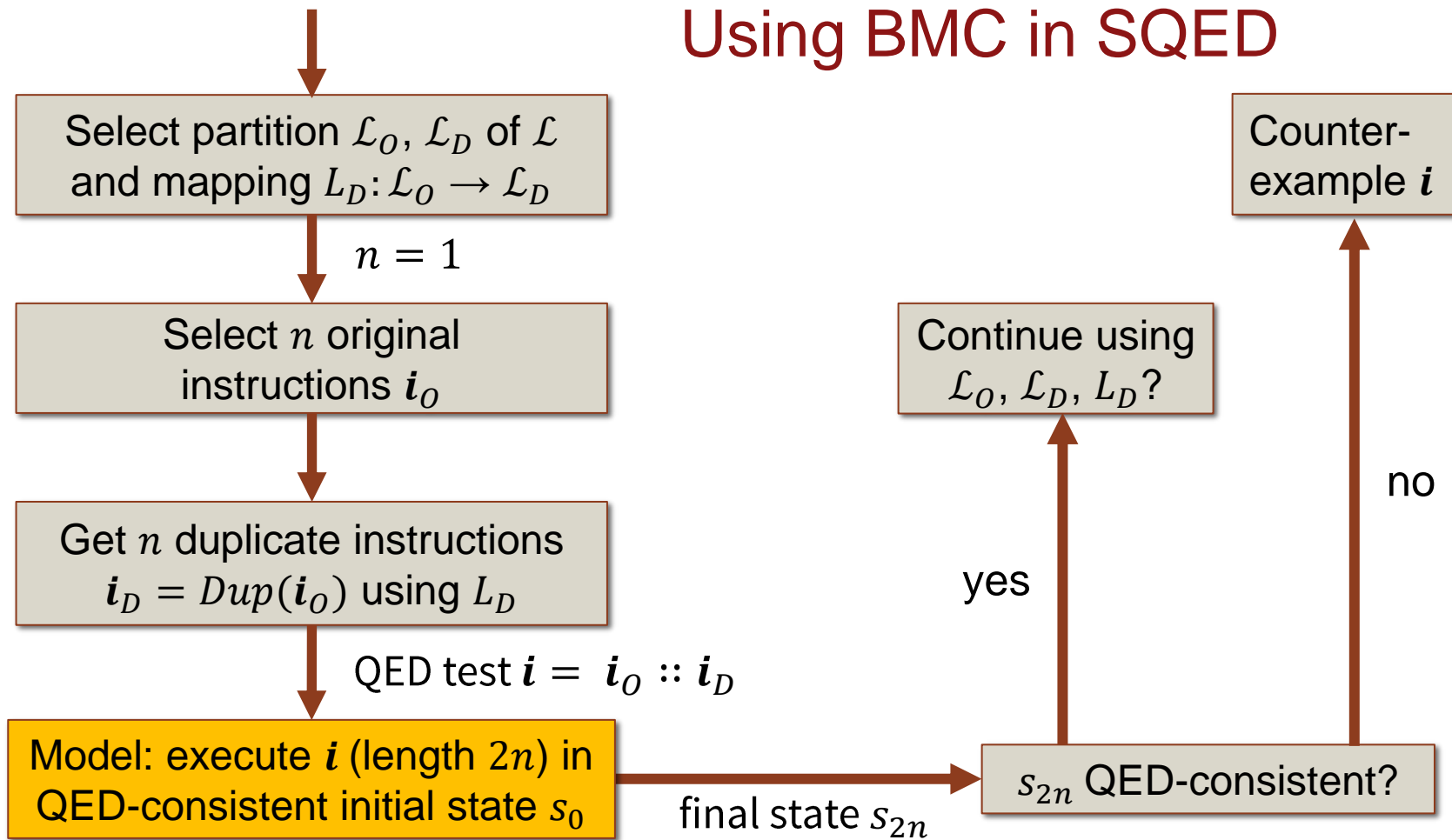
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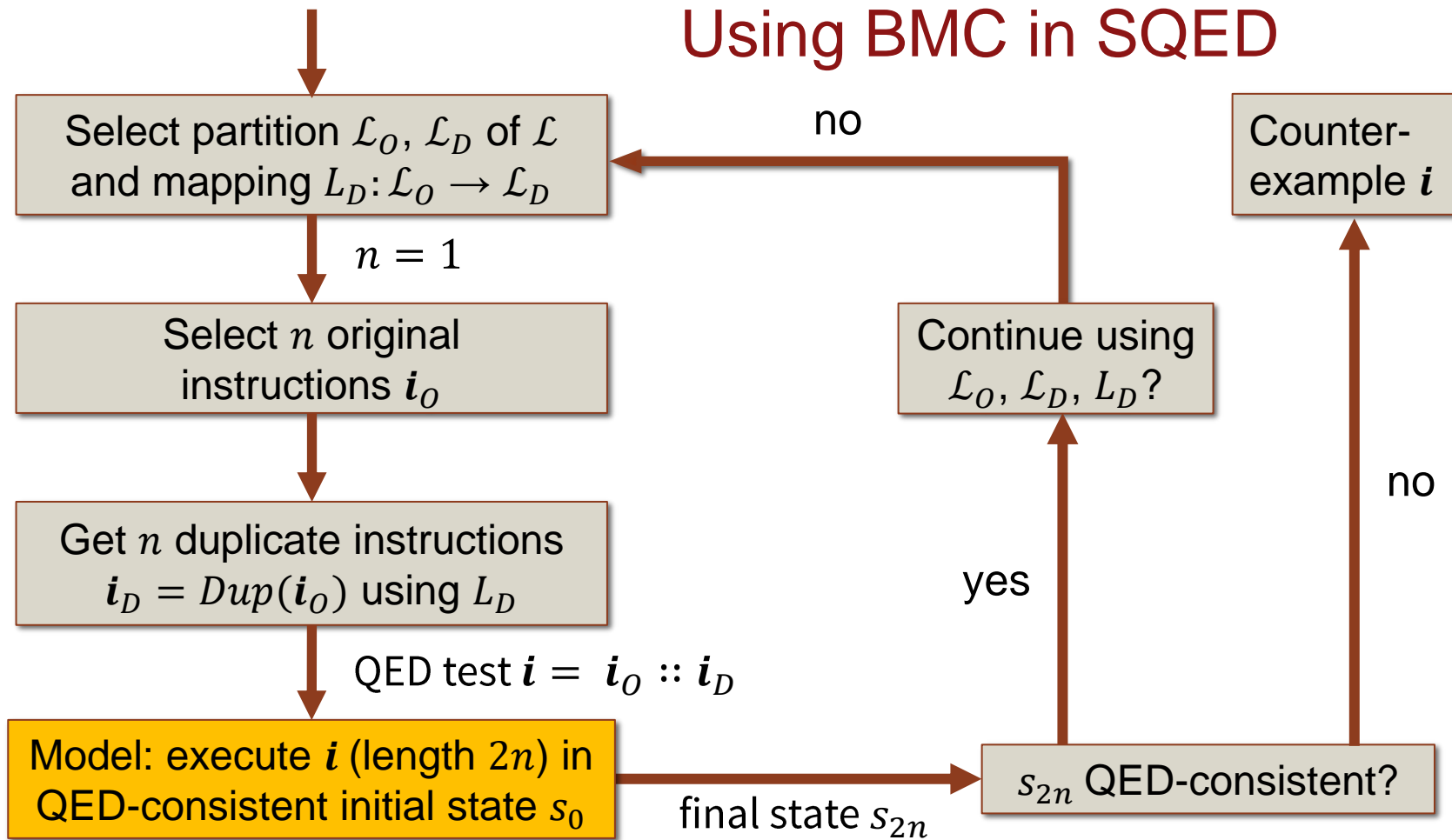
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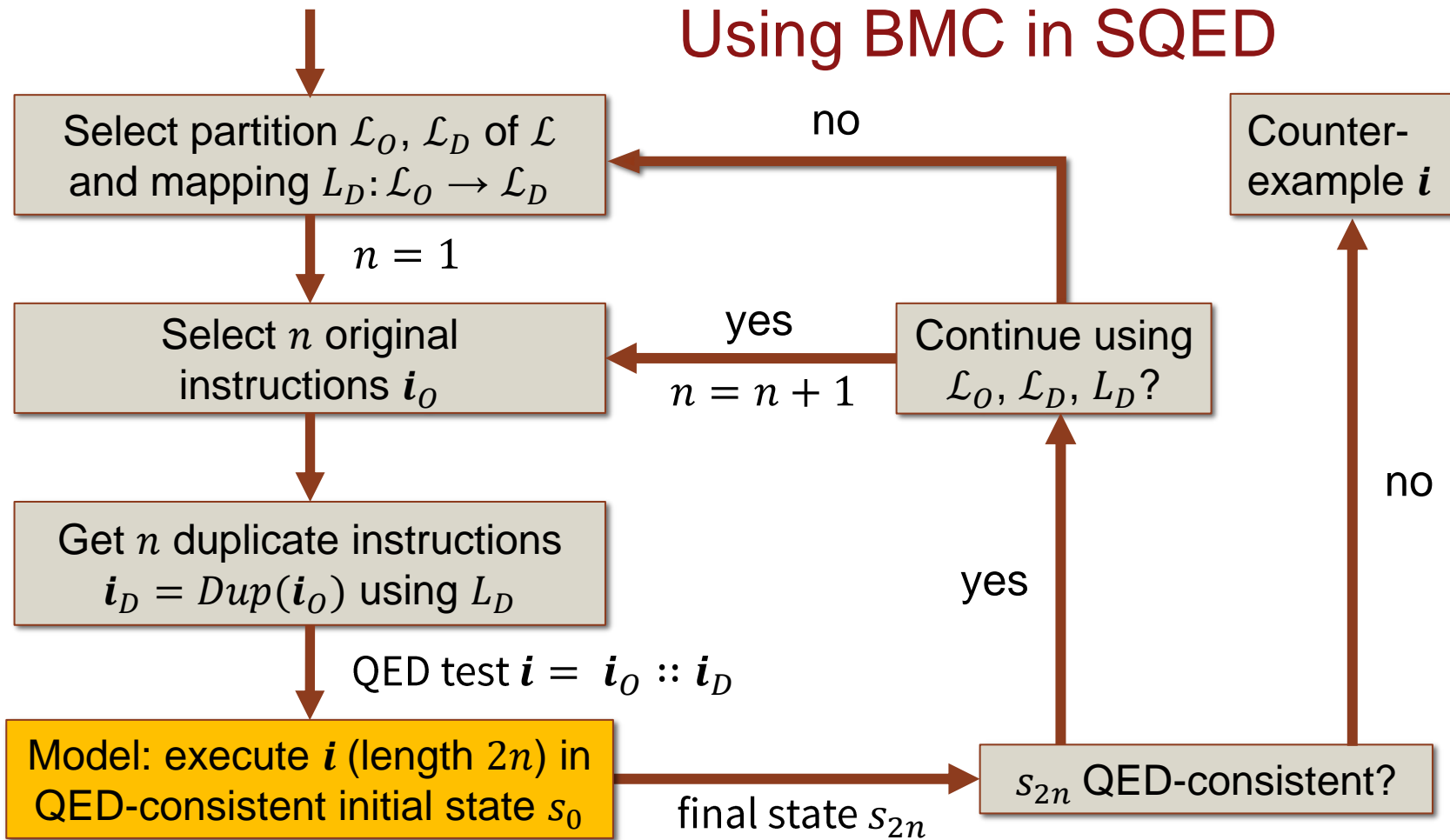


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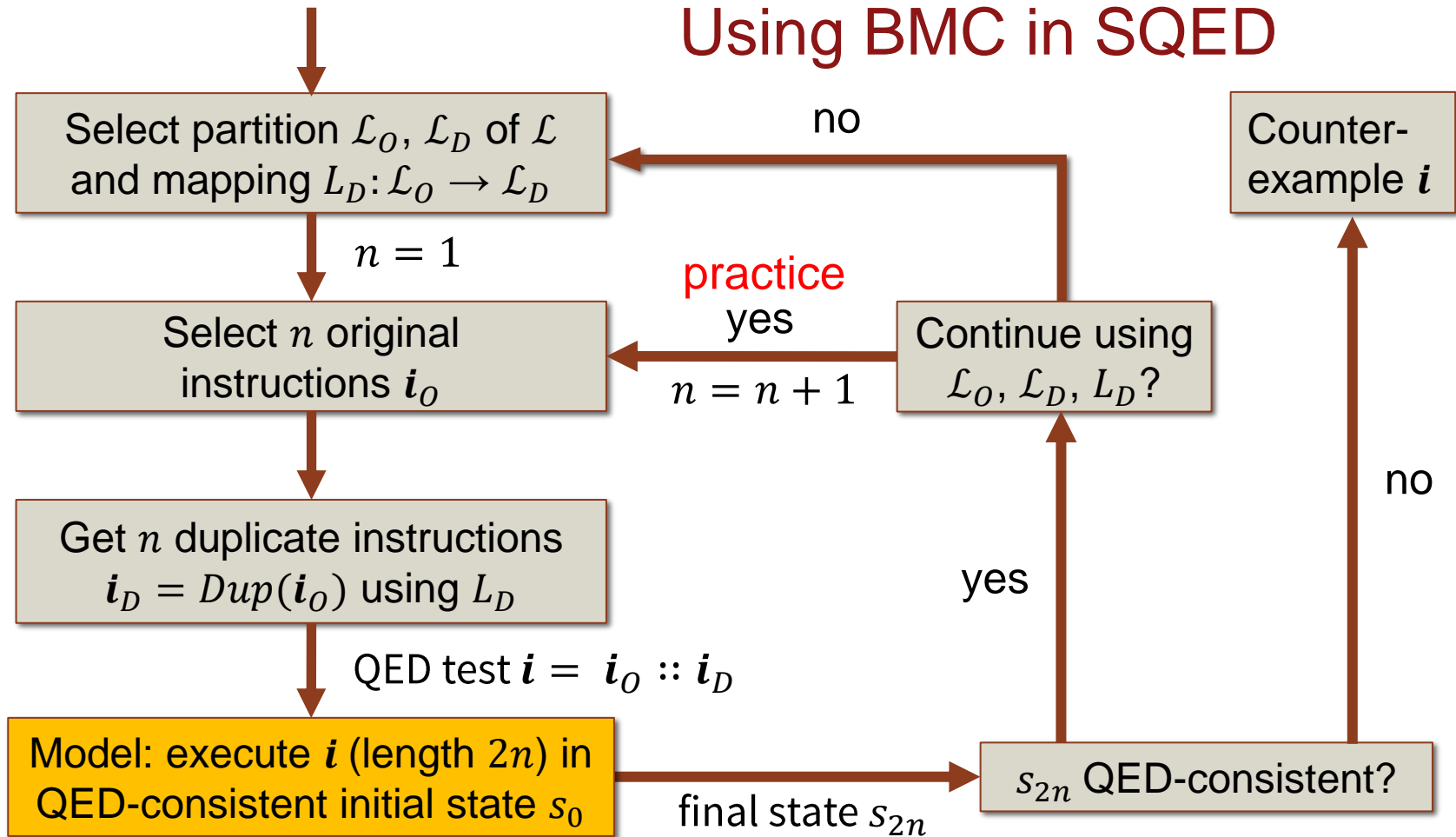




# Using BMC in SQED



# Using BMC in SQED



# Abstract Specification Relation

$$\begin{aligned} & \forall s, s' \in S, i \in I. \text{Spec}(s, i, s') \leftrightarrow \forall l \in \mathcal{L}. \\ & (l \neq \text{LocOut}(i) \rightarrow s(l) = s'(l)) \wedge \\ & (l = \text{LocOut}(i) \rightarrow s'(l) = \text{SpecOut}(i, s(\text{LocIn}(i)))) \end{aligned}$$

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- Transition  $T(s, i) = s'$  according to  $\text{Spec} \subseteq S \times I \times S$  iff:

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    - Output specification:  $\text{SpecOut}: I \times \mathcal{V}^2 \rightarrow \mathcal{V}$ .

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- Abstract spec needed **only for theory**, not practice.

# Bugs and Processor Correctness

Processor P is **correct** wrt. Spec:

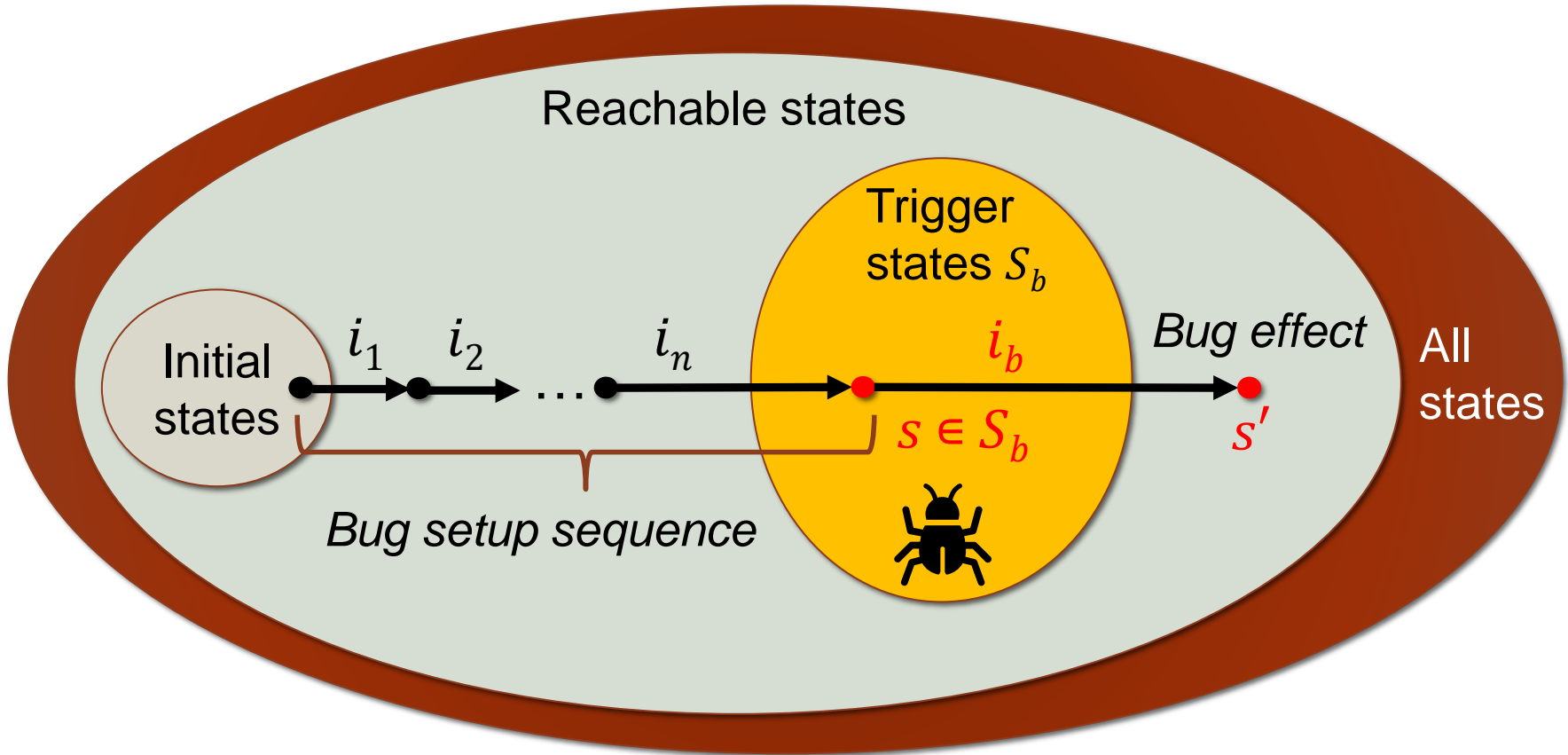
- $\forall s \in S, i \in I. reach(s) \rightarrow Spec(s, i, T(s, i)).$
- All instructions execute correctly in all **reachable** states.

**Bug:**

- Instruction  $i_b$  and set  $S_b \subseteq S$  of **bug-triggering states**.
- $S_b = \{s \in S \mid reach(s) \wedge \sim Spec(s, i_b, T(s, i_b))\}$



# Bug Triggering



# Single-Instruction Bugs and Correctness

Processor P is **single-instruction (SI) correct** wrt. Spec:

- $\forall s \in \mathit{Init}, i \in I. \text{Spec}(s, i, T(s, i)).$
- All instructions execute correctly in all **initial states  $\mathit{Init}$ .**

# Single-Instruction Bugs and Correctness

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**Single-instruction (SI) bug:**

- $\exists s \in Init, i \in I. \sim Spec(s, i, T(s, i)).$
- No setup sequence, well-studied approaches to checking.

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Single-instruction (SI) bug:

- $\exists s \in Init, i \in I. \sim Spec(s, i, T(s, i))$ .
- No setup sequence, well-studied approaches to checking.



Assumption:  $P$  is SI-correct.

# Soundness of SQED

$$QEDcons(s_0) \Rightarrow QEDcons(s_{2n})?$$

Initial  
states

$s_0$

$i_1$

$i_n$

$i_1'$

$i_n'$

$s_{2n}$

All  
states

$i_0$

$i_D$

**QED test  $i = i_0 :: i_D$**

# Soundness of SQED

**Theorem:** if  $\sim QEDcons(s_{2n})$  then processor  $P$  has a bug.

Initial states

$s_0$

$i_1$

$\dots$

$i_n$

$i_1'$

$\dots$

$i_n'$

$s_{2n}$

All states

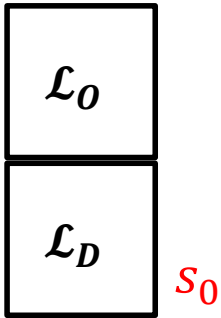
$i_0$

$i_D$

**QED test  $i = i_0 :: i_D$**

# Towards Completeness: Bug-Specific QED Test

$QEDcons(s_0)$



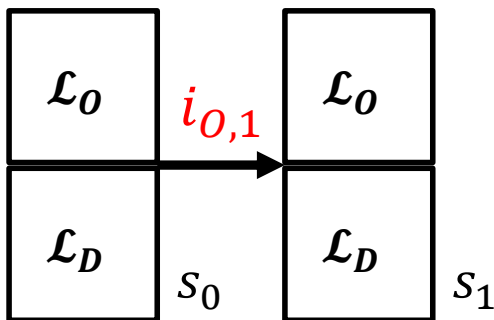
Initial state

QED test  $\mathbf{i} = (i_{O,1}, \dots, i_{O,n}) :: (i_{D,1}, \dots, i_{D,n})$  for some  $L_D$ .

- Flexibility in choosing  $L_D$ .
- QED-consistent **initial state**  $s_0$ .

# Towards Completeness: Bug-Specific QED Test

$QEDcons(s_0)$



Initial state

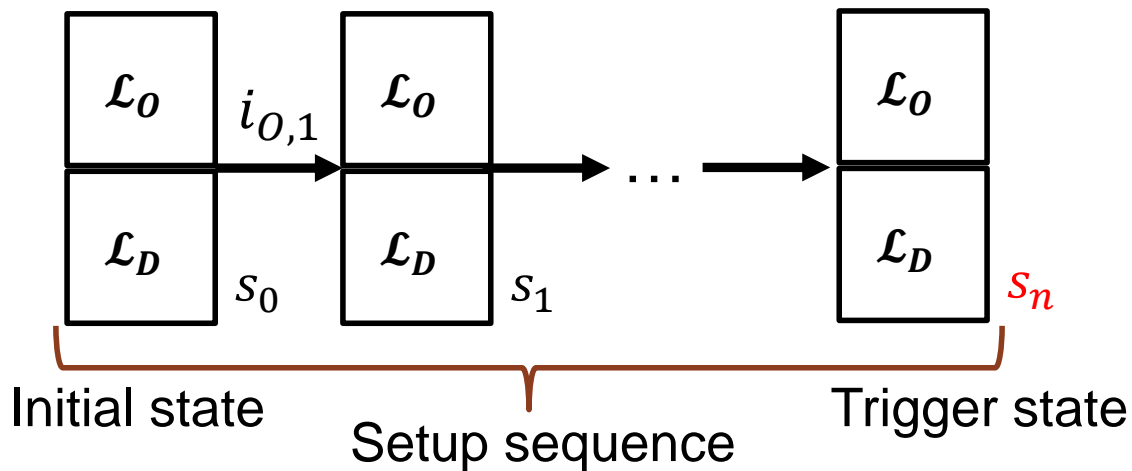
QED test  $\mathbf{i} = (i_{O,1}, \dots, i_{O,n}) :: (i_{D,1}, \dots, i_{D,n})$  for some  $L_D$ .

- QED-consistent initial state  $s_0$ .
- Let  $i_b = Dup(i_{O,1})$ :  $i_{O,1}$  meets  $Spec$  due to SI-correctness.



# Towards Completeness: Bug-Specific QED Test

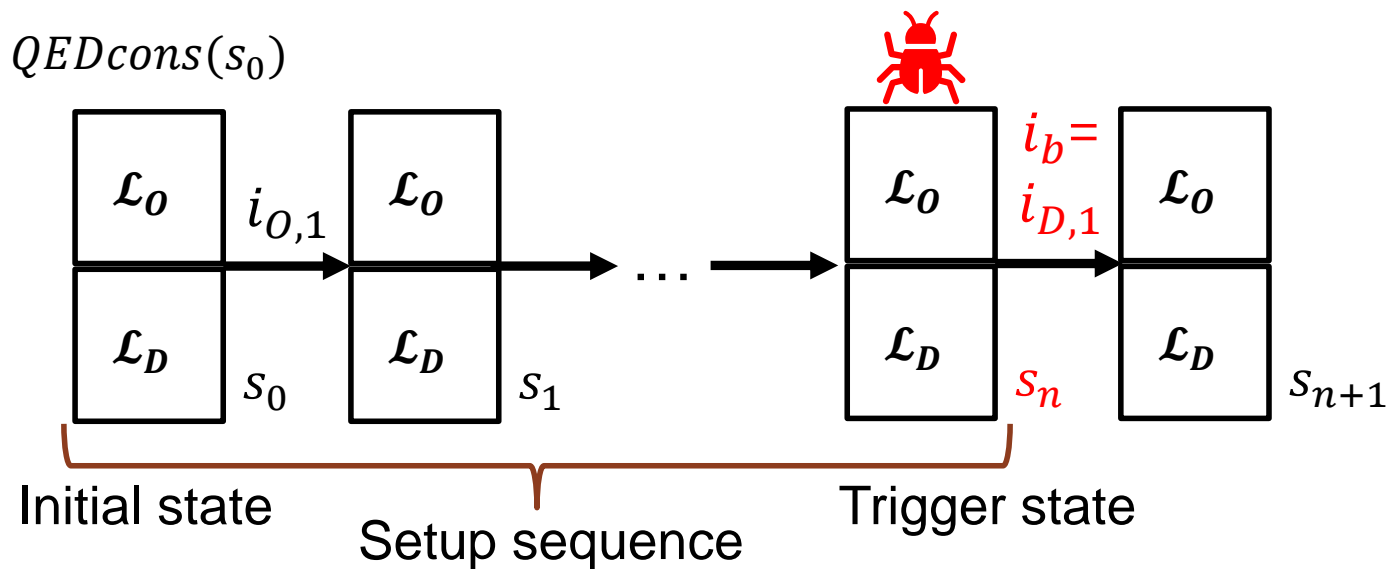
$QEDcons(s_0)$



QED test  $\mathbf{i} = (i_{O,1}, \dots, i_{O,n}) :: (i_{D,1}, \dots, i_{D,n})$  for some  $L_D$ .

- Setup sequence  $i_{O,1}, \dots, i_{O,n}$  to reach triggering state  $s_n \in S_b$ .

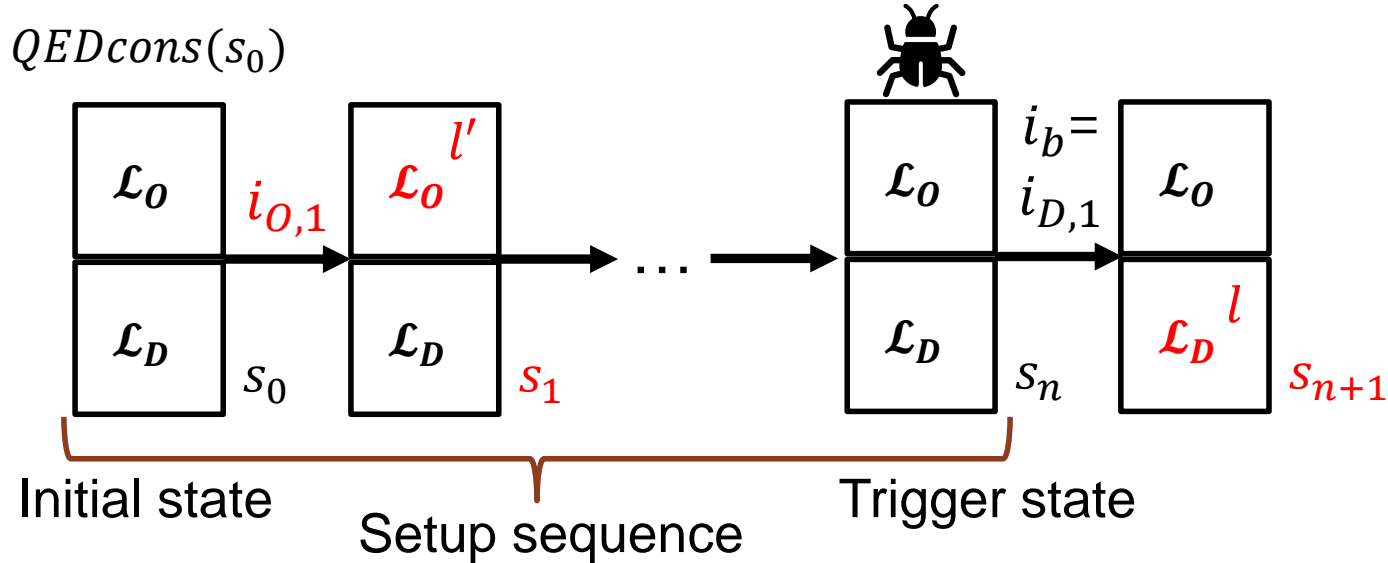
# Towards Completeness: Bug-Specific QED Test



QED test  $\mathbf{i} = (i_{O,1}, \dots, i_{O,n}) :: (i_{D,1}, \dots, i_{D,n})$  for some  $L_D$ .

- Setup sequence  $i_{O,1}, \dots, i_{O,n}$  to reach triggering state  $s_n \in S_b$ .
- Bug instruction  $i_b = Dup(i_{O,1})$  fails in  $s_n$ .

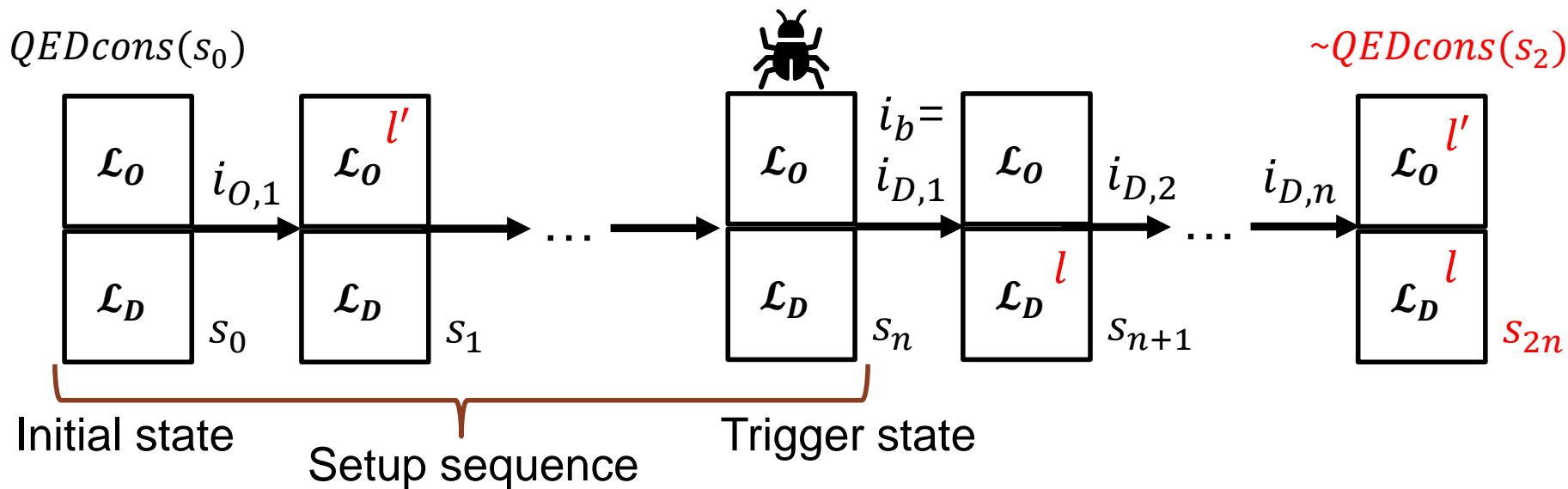
# Towards Completeness: Bug-Specific QED Test



QED test  $\mathbf{i} = (i_{O,1}, \dots, i_{O,n}) :: (i_{D,1}, \dots, i_{D,n})$  for some  $L_D$ .

- E.g. wrong value at output location  $l$  of  $i_b$  in  $s_{n+1}$ .
- Correct value at original output location  $l'$  of  $i_{O,1}$  in  $s_1$ .

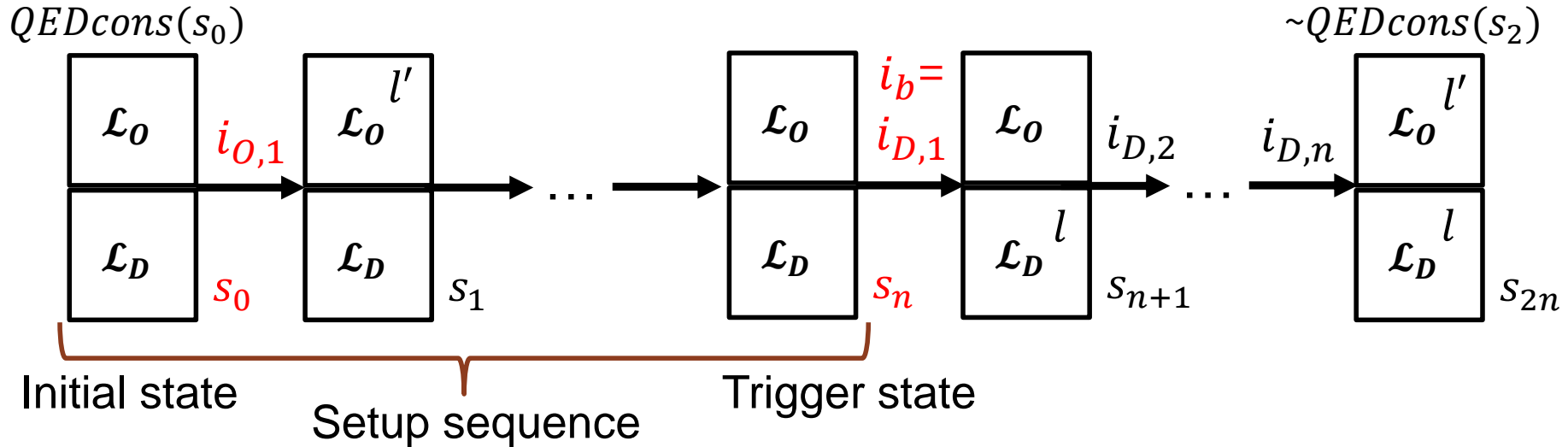
# Towards Completeness: Bug-Specific QED Test



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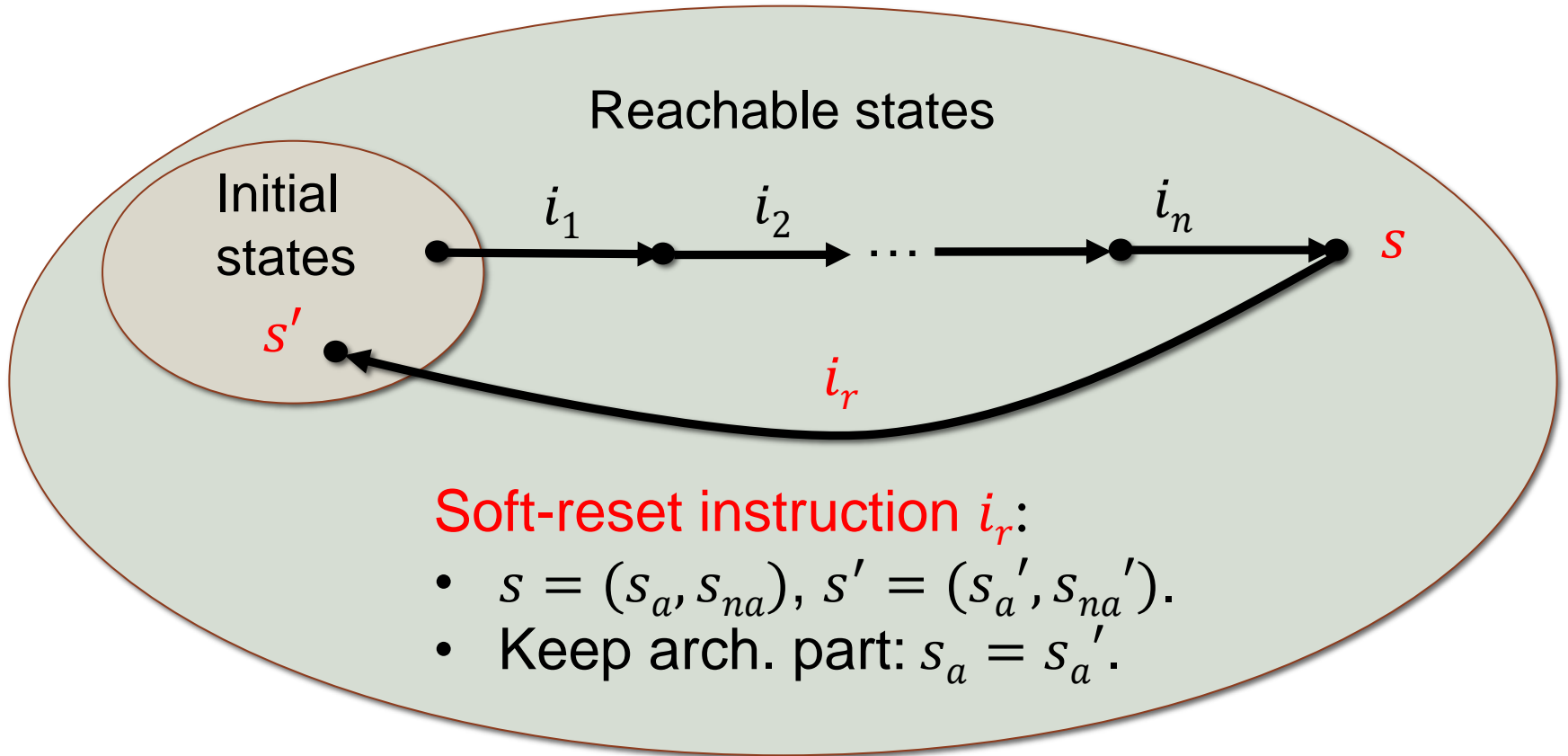
- Mismatching values at locations  $l$  and  $l'$  in  $s_{2n}$ .
- Final state  $s_{2n}$  **QED-inconsistent**.

# Conditional Completeness

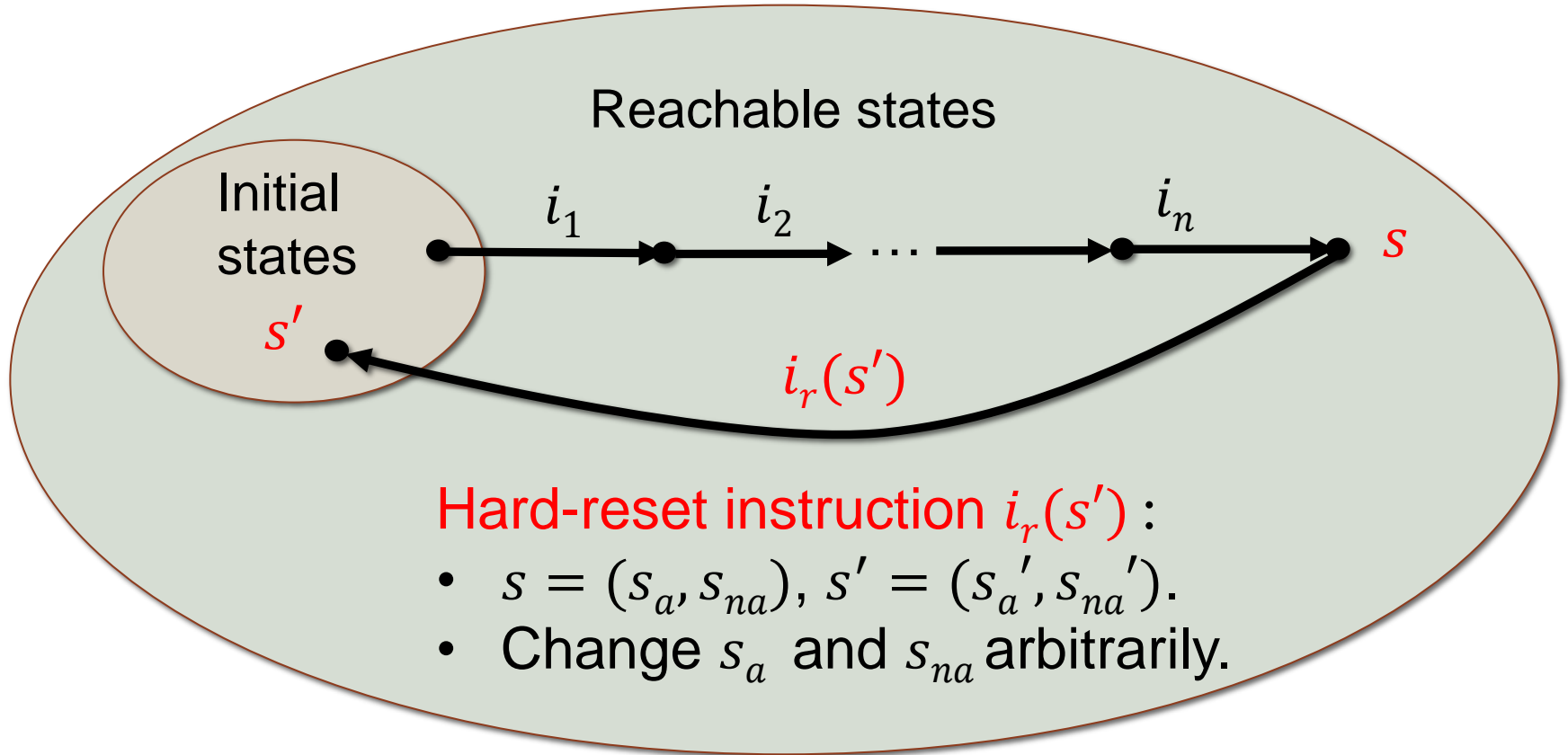


**Theorem:** *if a bug-specific QED test  $i$  exists, then  $i$  fails.*

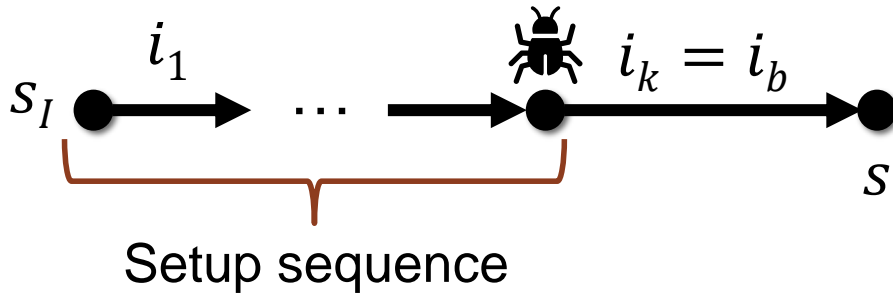
# Extensions: Reset Instructions



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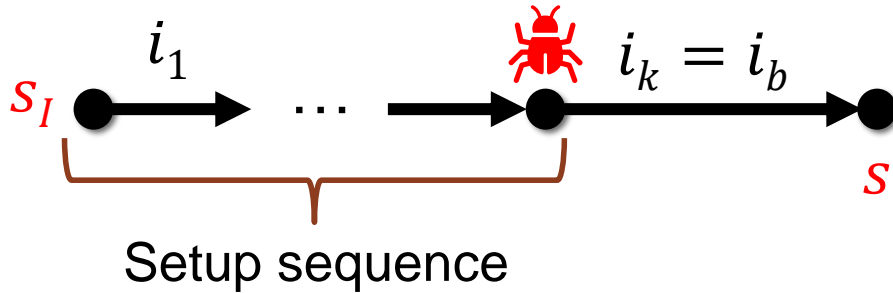
# Extensions: QED Test with Reset



- Bug set up and triggered by  $i_1, \dots, i_k = i_b$ .
- **No duplication**: check states after  $i_k = i_b$  with(out) reset.

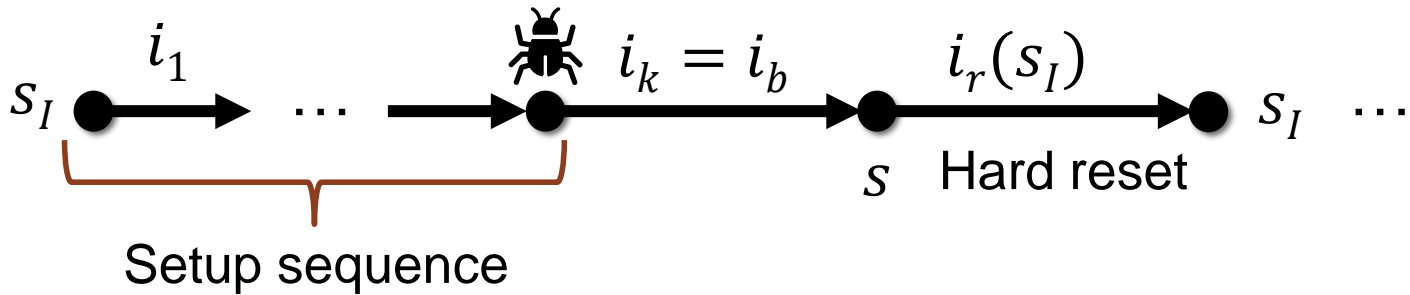


# Extensions: QED Test with Reset



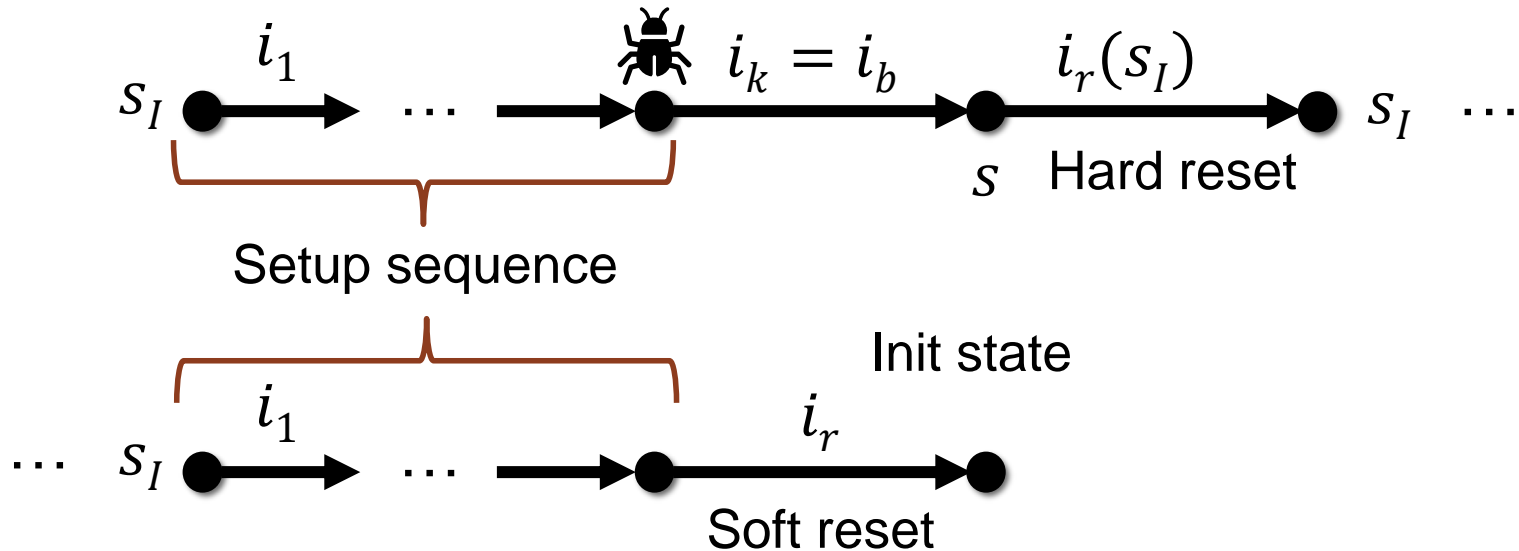
- Bug set up and triggered by  $i_1, \dots, i_k = i_b$ .
- Execute  $i_1, \dots, i_k = i_b$  from  $s_I \in \text{Init}$ : wrong value in state  $s$ .

# Extensions: QED Test with Reset



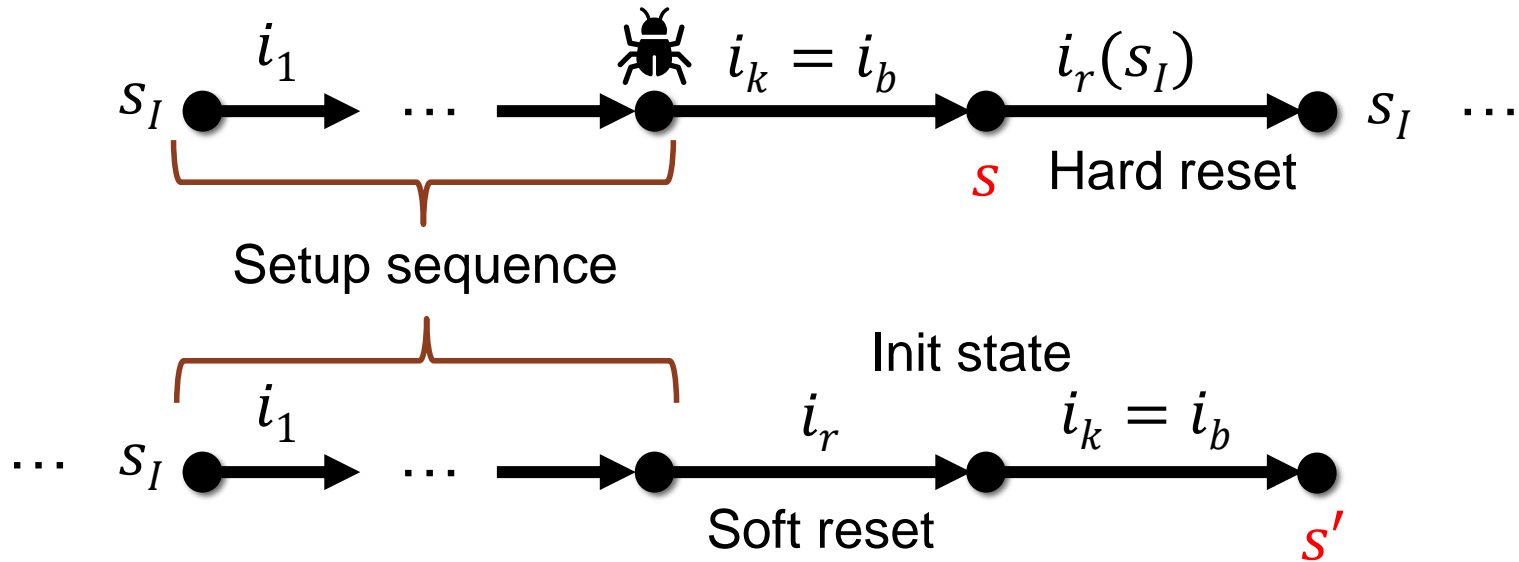
- Execute hard reset in state  $s$ , get back to  $s_I$ .
- Idea: execute  $i_1, \dots, i_k = i_b$  again with soft reset before  $i_b$ .

# Extensions: QED Test with Reset



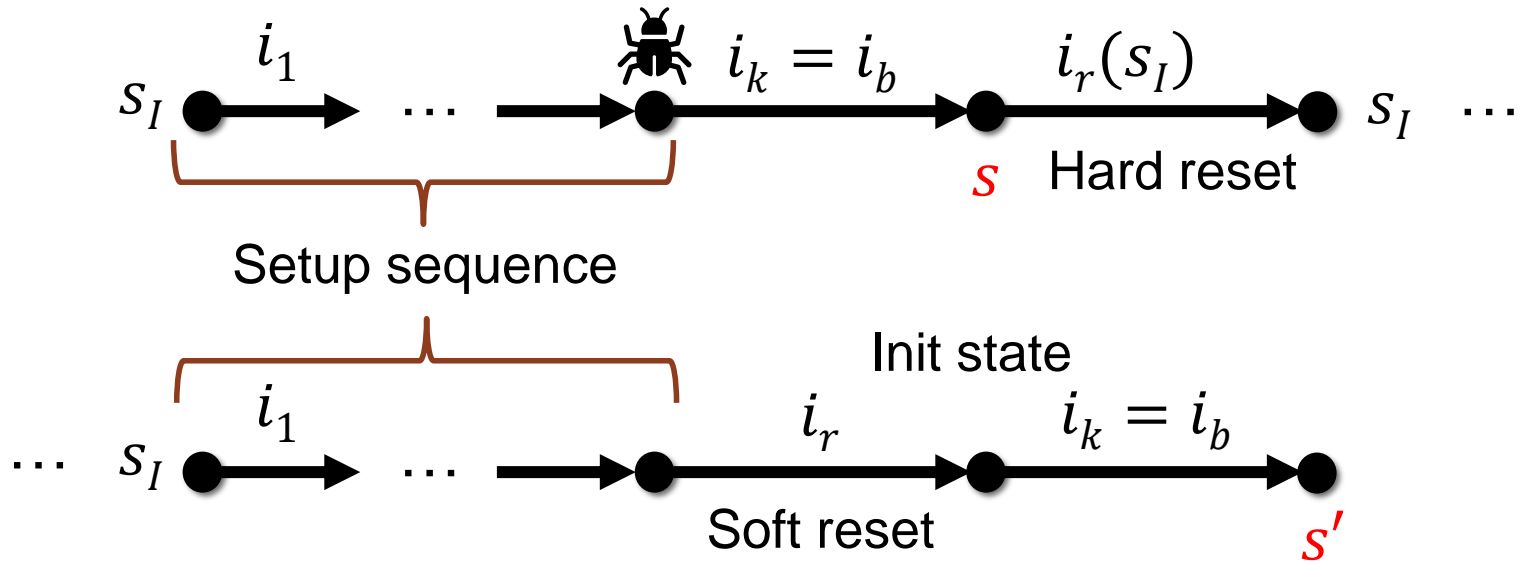
- Execute soft reset in bug-triggering state before  $i_k = i_b$ .
- Make use of SI correctness.

# Extensions: QED Test with Reset



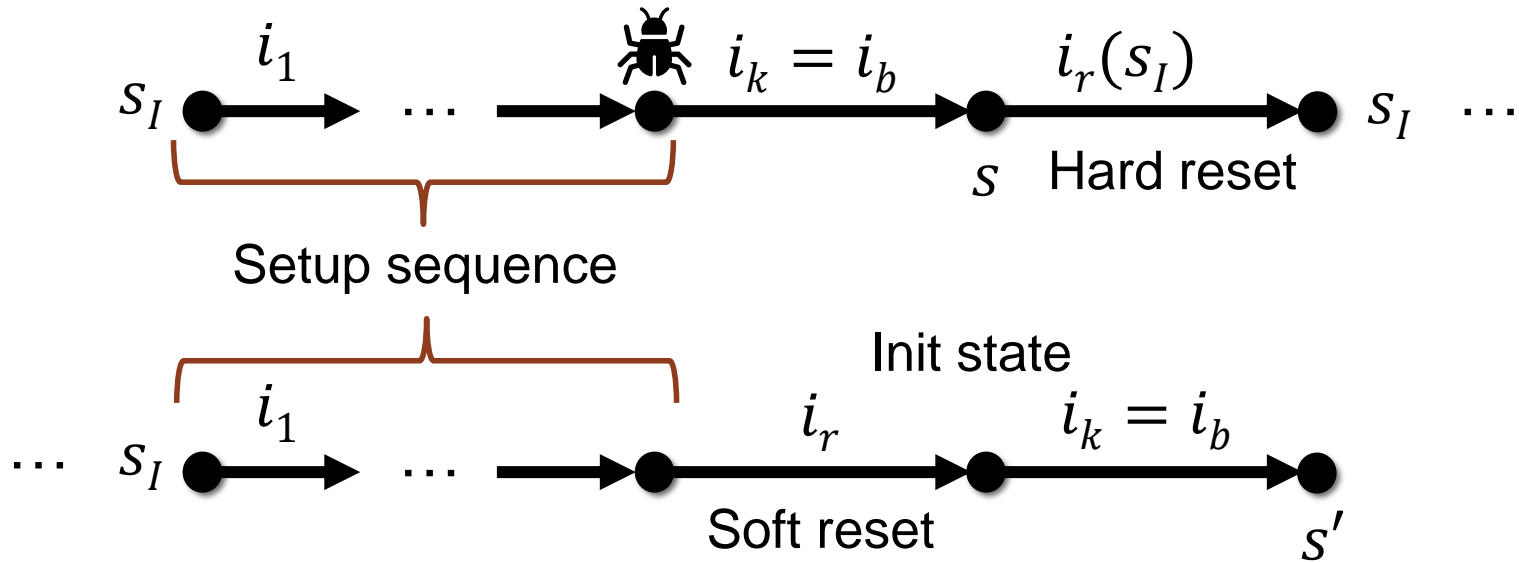
- Bug instruction  $i_k = i_b$  executes correctly.
- Compare  $s$  and final state  $s'$ .

# Extensions: QED Test with Reset



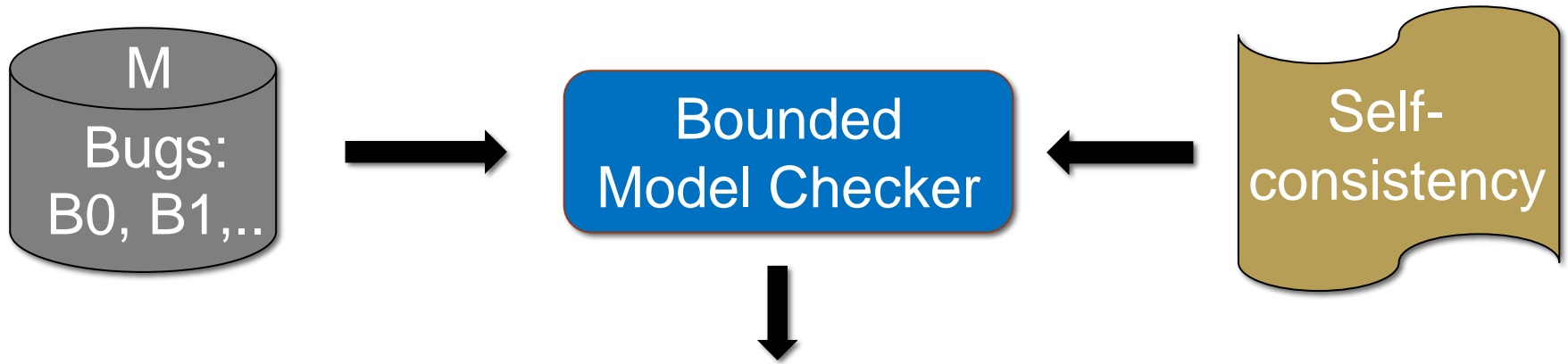
- QED test with reset fails iff  $s(l) \neq s'(l)$  for a location  $l$ .

# Extensions: QED Test with Reset



**Theorem (full completeness):** *if  $P$  is SI correct and has no failing QED test with reset, then  $P$  is correct.*

# Summary: SQED Soundness and Completeness

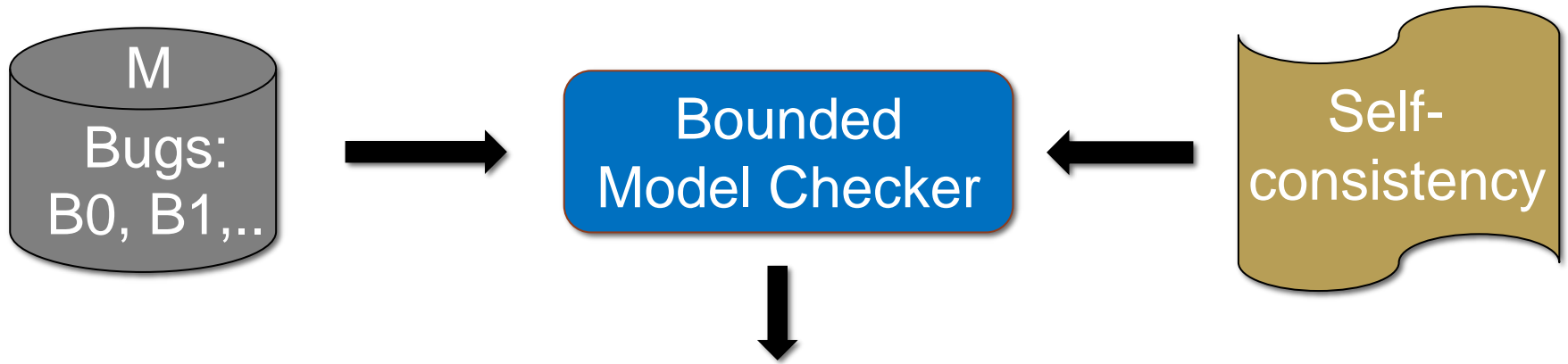


Does  $P \in Spec$  hold in  $M$ ?  $\approx$  Is  $M$  self-consistent?

- If  $M$  not self-consistent then  $B \in M$ .
- Self-consistency covers bug  $B$ .

No spurious cex

# Summary: SQED Soundness and Completeness



Does  $P \in Spec$  hold in  $M$ ?  $\approx$  Is  $M$  self-consistent?

- If  $B \in M$  then  $M$  not self-consistent.
- Self-consistency covers bug  $B$ .

Conditional/full  
Completeness



# Future Work

Leveraging QED test extensions:

- Soft/hard reset not yet applied in practice.
- Design-for-verification approach.

Formal model refinements:

- Instruction pipelines, multiprocessor systems.
- Deadlock detection.
- Symbolic starting states.

*Thank you for your attention!*